Dynamic Response Shimmy Analysis of Nose Landing Gear Using a Linear and a Non-Linear Mathematical Model

Aakash S. Agarwal

Department of Mechanical Engineering, MIT College of Engineering, Kothrud, Pune, Maharashtra 411038, India

Abstract - Landing gear is undoubtedly the most complex structure in the design of an aircraft. Even after years of study, Shimmy oscillations are still a problem in the design and operation of an aircraft. In the present work, Mathematical models are developed and simulated in MATLAB to study the dynamic response of a nose landing gear to understand the concept of shimmy in an easier way. Firstly a linear model of nose landing gear is developed and the time histories of the vertical displacements are simulated. This simulation proves very effective to understand the problem of shimmy if the parameters are considered to be linear in behavior The results are also validated. Further, to convert this linear relations to non linear ones(which are more realistic to occur) , a non linear mathematical model is developed. Here the stability boundaries are simulated to understand the regions of stability. Also the stability boundaries are analyzed for damping parameters which are directly related to the intensity of the oscillations. Relaxation length and vertical force, two of the most important parameters are varied and their stability regions are plotted.

Keywords :shimmy vibrations, mathematical models, linear model,

I. INTRODUCTION

Shimmy is the self-excited violent vibrations of the aircraft Landing gear with respect to the runway surface. It is often describes as a conversion of the kinetic energy resulting from the forward motion of the aircraft vehicle during landing or taxiing. In some cases it may be due to the resonance when the vertical oscillations of the landing gear equals the natural frequency of vibrations of the aircraft[1]. As such, there are numerous ways for studying this complex shimmy phenomenon, some of which are very complex(models of hundreds of DOF) and hence require tons of calculations.

Shimmy being complex and nonlinear in nature, is often very difficult to explain in terms of certain pre-decided parameters. Torsional and lateral bending interacts with the landing gear shimmy [2], thus leading to the study of bifurcation diagrams. Geometric coupling and mechanical freeplay [3] also plays a major role in shimmy whereas a commonly neglected parameter of dry friction [4][5] also adds to the shimmy phenomenon. Owing to such diversity in parametric effects, various methods are used for analysis and simulation. Developing mathematical models thus proves to be an effective way to understand and analyze this phenomenon quite easily.

In this present work, two simple mathematical models are considered for the study and analysis of this shimmy phenomenon .The first model is a linear model for a simple nose landing gear in which the time histories for vertical displacements are simulated. The effect of spring and damper system is studied in this case. In the second model a nonlinear mathematical shimmy model is considered which also takes into account the effect of lateral and torsional vibrations and the various effects of tire deformation. The stability boundaries are simulated for various parameters to understand the effect of varying damping constant on the stability of the landing gear.

II. LINEAR MATHEMATICAL MODEL

A linear generic mathematical model is being considered to study the behavior of Shimmy Vibrations on a generalized aircraft. Mass M is the mass of the entire aircraft whereas mass m is the mass of the landing gear system. A suspension system is developed in the model with the help of a spring and a damper. A suspension system with spring k1 and damper c1 is considered between the landing gear system and a suspension with spring k2 and damper c2 between the aircraft(including the fuselage) and the landing gear system.

The results are simulated without the shimmy dampers and hence the value of c2 is 0.Hence damper c2 is not considered while formulating the equations for the same.The results are verified as in [6].

Parameter	Value	Units
Mass of tire, collar, damper fluid , fuselage, M	2000	Kg
Mass of landing gear, m	30	kg
Stiffness of suspension spring 1, k1	1750000	N/m
Stiffness of spring 2, k2	651000	N/m
Initial displacement, x_0	0	M
Damping coefficient, c_1	1110	N/m

Acceleration due to gravity, g	9.81	m/s^2
Displacement of masses M and m, $x_1(t)$, $x_2(t)$	-	m

Table 1.Values of various Parameters used for the Linear Model

To solve the differential equations, the initial conditions are as follows-

$$\square_1(0) = \square_2(0) = 0.1\ \square$$

$$\dot{\square}_1(0) = \dot{\square}_2(0) = 0.2\ \square/\square$$

ANALYTICAL MODELING

CASE 1

In this case, the base, i.e. the runway is considered to be passive and hence no surface irregularities are considered while simulating the model.

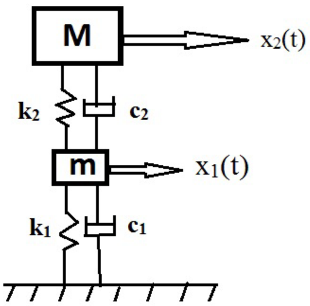

Figure 1. Linear Mathematical Model

Equation of motion for the same is given by-

$$\square\ddot{\square}_\square + \square_\square(\dot{\square}_\square - \dot{\square}_\square) + (\square_\square + \square_\square)\square_\square - \square_\square\square_\square = \square$$

$$\square\ddot{\square}_\square + \square_\square(\dot{\square}_\square - \dot{\square}_\square) - \square_\square\square_\square + \square_\square\square_\square = \square\square$$

The complete solution for the system of equations is given by-

$$\square_\square(\square) = \square.\square\square\square\square\square^{-\square.\square\square\square}\square\square\square(\square\square.\square\square\square - \square.\square\square\square) + \square.\square\square\square\square\square^{-\square\square.\square\square\square}\square\square\square(\square\square\square.\square\square\square - \square.\square\square\square\square) + \square.\square\square\square\square$$

$$\square_\square(\square) = \square.\square\square\square^{-\square.\square\square\square}\square\square\square(\square\square.\square\square\square - \square.\square\square\square) + \square.\square\square\square\square\square\square^{-\square\square.\square\square\square}\square\square\square(\square\square\square.\square\square\square - \square.\square\square\square\square) + \square.\square\square\square\square$$

The results are simulated in MATLAB as follows. $x_1(t)$ shows the vibrations in the landing gear-base vertical plane whereas $x_2(t)$ shows the vibrations in the main aircraft vertical plane.

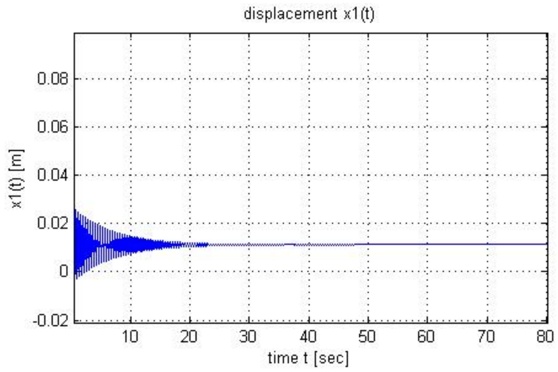

Figure 2.Time history of displacement $x_1(t)$

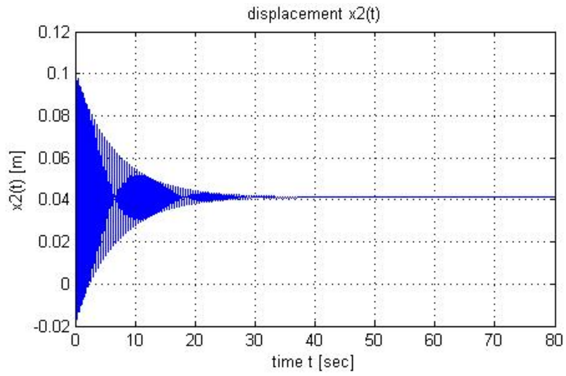

Figure 3.Time history of displacement $x_2(t)$

CASE 2

In this case the base surface is excited in the model, i.e. the irregularities of the runway are taken into account as follows-

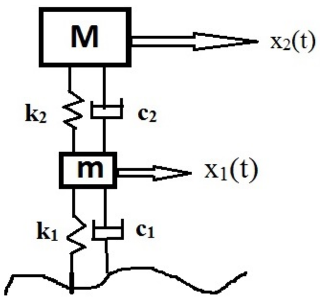

Figure 4.Linear Mathematical Model with Base Excitation

The equation of motion for this model is

$$\square\ddot{\square}_\square + \square_\square(\dot{\square}_\square - \dot{\square}_\square) + (\square_\square + \square_\square)\square_\square - \square_\square\square_\square = \square_\square\square\square\square\square\square$$

$$\square\ddot{\square}_\square + \square_\square(\dot{\square}_\square - \dot{\square}_\square) - \square_\square\square_\square + \square_\square\square_\square = \square\square$$

The values are same as that of the previous case and the harmonic excitation is assumed to be 0.2sin10t.

The complete solution for this system of equations is

□_□(□)
= □.□□□□^(−□.□□□) □□□(□□.□□□ − □.□□□)
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The simulation results are as follows

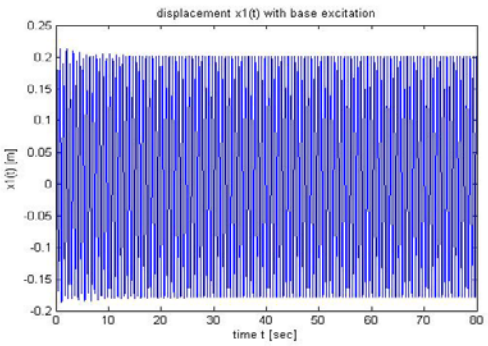

Figure 5.Time History of Displacement $x_1(t)$ with base excitation

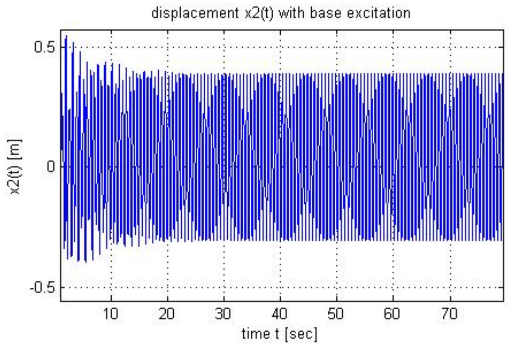

Figure 6.Time history of displacement $x_2(t)$ with base excitation

III. NON LINEAR MATHEMATICAL MODEL

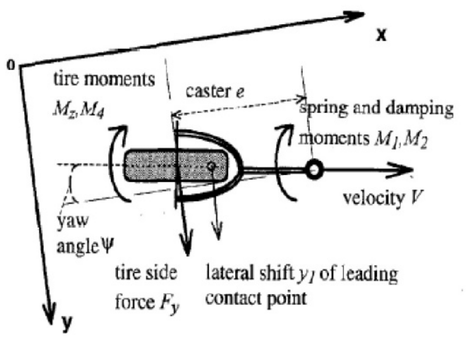

Figure 7.Top view of Non- Linear Mathematical Model

The nonlinear mathematical shimmy model, as shown in figure, consists of the torsional dynamics of the landing gear, the forces and the moments, and of approximations to describe the tire's elastic lateral qualities. The figure for the same is from [8]

Table 2.Values of various parameters used in Non-linear Model.

Parameter	Value	Unit
Half contact length a	0.1	m
Caster length e	0.1	m
Moment of Inertia I_Z	1.0	Kg m^2
Torsional spring rate c	-100000	Nm/rad
Side force derivative $c_{F\alpha}$	20	1/rad
Moment derivative $c_{M\alpha}$	-2	m/rad
Tread width moment constant κ	-270	Nm^2/rad
Relaxation length σ=3*a	0.3	m
Vertical force F_Z	9000	N
Torsional damping constant k	0-(-50)	Nm/rad/s
Velocity V	0-80	m/s

The stability boundary in (-k)-V plane with the parameter set p3 represents the case for varying parameters during landing. It is quadratic in k as follows:

□_1 □^2 + □_2 □ + □_3 = 0

Where coefficients b_1, b_2, b_3 are functions of (V,p) as follows-

□_1 = □^2 □

□_2 = □^3 □_□ − □_□□_{□□}□□□ + 2□□□ − □^2 □□
+ □_□□_{□□}□□□ − □_□□□_{□□}□□□
+ □_□□^2 □_{□□}□□

□_3 = (□_□□□_□□_{□□} − □_□□□_□□_{□□} + □□_□
− □_□□□_□□□_{□□} + □_□□_{□□}□□_□
− □_□□□_{□□}□□_□ + □_□□^2 □_{□□}□_□) □^2
+ □_□□_{□□}□□□ − □_□□□_{□□}□□□
+ □□^2 − □^2 □□ − □_□□_{□□}□□□
+ □_□□^2 □_{□□}□□

The equation have two solutions, both are valid but one of branches is positive and thus is physically meaningless value of k.

IV. SIMULATION RESULTS

In figure 8, relaxation length σ is varied in k-V plane to study its effect. For small values of σ (σ<0.1m), the instability is more at smaller velocities and the stability is more at large velocities. Whereas for relaxation length σ (σ>0.1m), the stability is more at smaller velocities and the instability is more at larger velocities.

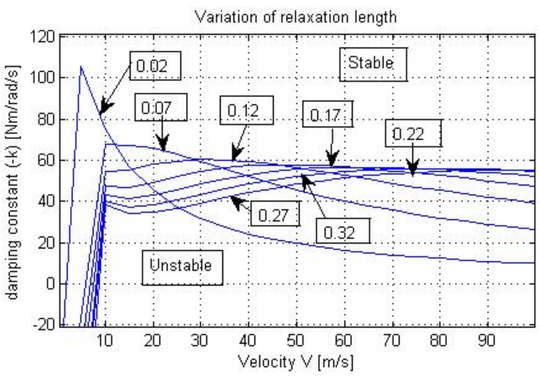

Figure 8.Variation of Damping Constant (-k) with velocity V for different values of relaxation length σ

In figure 9 , the vertical force FZ is varied in the k-V plane to study the effect of load on stability region. This variation of vertical force is important because values of FZ may change due to weight, acceleration, and braking during landing and taxiing. As the force and vertical velocity V increase, a larger value of k is needed for stability. From the simulation results, below V=16 m/s, there is no instability for negative damping coefficients.

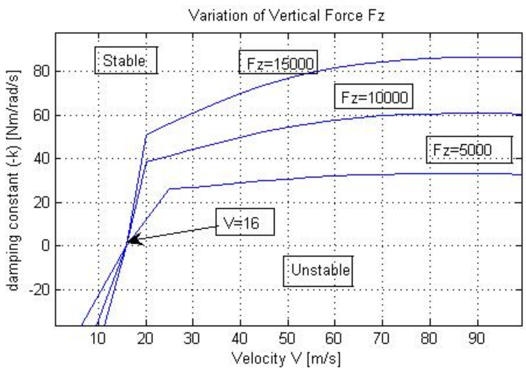

Figure 9.Variation of Damping constant (-k) with velocity V for different values of Vertical Force F_Z.

V. CONCLUSION

Two different mathematical models were developed to study the effect of the key parameters on shimmy of aircraft Landing gears. Different analytical methods were applied to study these mathematical models.

Firstly, a simple linear mathematical model was developed to study the vibrations in the vertical-Y direction. Two cases were considered, a model with simple landing and a model with base excitation. The time histories of vertical displacements at the runway-landing gear and landing gear-main aircraft were simulated in MATLAB for both the cases. With base excitation the shimmy vibrations were more and hence this plays a very important part in shimmy analysis.

The main purpose of using the linear model was that it reduces the complexity of the system and hence makes it easy to analyze and simulate the behavior. But in actual practice, the behavior of the landing gear is nonlinear. It is observed that the instability in the linear model causes shimmy in nonlinear model.

A nonlinear model was considered to study the actual dynamic response of the same. The focus of this model was to study the effect of variation of damping constant (-k) on various parameters such as velocity, relaxation length of tire deflection σ , and vertical force FZ. The results were simulated in MATLAB and were validated as in [8] .

For different relaxation lengths σ, it was observed that small values of σ cause more stability at small velocities and more stability at larger velocities. Whereas large values of σ causes more stability at low velocities. For variation in vertical force FZ, it was observed that larger values of FZ and V require larger value of (-k) for stability. Larger values of (-k) will create instability for other parameters and hence is undesirable.

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