PSNR, SSIM and SSIM Index Map Enhancement by Image Denoising Using DTDWT and Wiener Filter

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Abstract— Wavelet techniques are successfully applied to various problems in signal and image processing. Denoising of images is an important task in image processing and analysis and it plays a significant role in modern applications in different fields, including medical imaging and preprocessing for computer vision. The goal of denoising is to remove the noise. The Discrete Wavelet Transform (DWT) of image signals produces a non-redundant image representation, which provides better spatial and spectral localization of image formation. But, it has lack of shift invariance, lack of symmetry of the mother wavelet and poor directional selectivity. To overcome this problem, Kingsbury proposed a dual-tree implementation of the CWT (DT-CWT), which uses two trees of real filters to generate the real and imaginary parts of the wavelet coefficients separately. The DT-CWT is an alternative to the basic DWT, the outputs of each tree are down sampled by summing the outputs of the two trees during reconstruction and the aliased components of the signal are suppressed and approximate shift invariance is achieved. Image denoising involves the manipulation of the image data to produce a visually high quality image. Selection of the denoising algorithm is application dependent. Hence, it is necessary to have knowledge about the noise present in the image so as to select the appropriate denoising algorithm. The wavelet based approach finds applications in denoising images corrupted with Gaussian noise. In the case where the noise characteristics are complex, the multifractal approach can be used. The objective of this work is the pre-processing of an image before using it in applications. The pre-processing is done by denoising of image. In order to achieve this, Dual Tree Discrete Wavelet Transform (DT-DWT) and a wiener filter are applied on images. The DT-DWT has an excellent performance in the denoise field and use of wiener filter can further enhance the quality of image by filtering out the noise. The performance of proposed system is analyzed on the basis of Peak Signal to Noise Ratio (PSNR), Structural Similarity Index (SSIM) and SSIM Index Map.

Keywords— Denoising, Dual Tree Discrete Wavelet Transform (DT-DWT), Wavelet Transform (WT), Wiener Filter, Peak Signal to Noise Ratio (PSNR), Structural Similarity Index (SSIM) and SSIM Index Map.

I. INTRODUCTION

Wavelet theory is one of the most modern areas of mathematics. Masterfully developed by French researchers, such as Yves Meyer, Stéphane Mallat and Albert Cohen, this theory, is now used as an analytical tool in most areas of technical research: mechanical, electronics, communications, computers, biology and medicine, astronomy and so on. In the field of signal and image processing, the main applications of wavelet theory are compression and denoising. The development of wavelet transforms over the last two decades revolutionized modern signal and image processing, especially in the field of image denoising. During the 1990s, the field was dominated by wavelet shrinkage and wavelet thresholding methods. From a historical point of view, wavelet analysis is a new method, though its mathematical underpinnings date back to the work of Joseph Fourier in the nineteenth century. Fourier laid the foundations with his theories of frequency analysis, which proved to be enormously important and influential. The attention of researchers gradually turned from frequency-based analysis to scale-based analysis when it started to become clear that an approach measuring average fluctuations at different scales might prove less sensitive to noise [3].

It is well known fact that signals do not exist without noise, which may be negligible (i.e. high SNR) under certain conditions. However, there are many cases in which the noise corrupts the signals in a significant manner, and it must be removed from the data in order to proceed with further data analysis. There is a wide range of applications in which denoising is important. Examples are medical image/signal analysis, data mining, radio astronomy and many more. Each application has its special requirements. For example, noise removal in medical signals requires specific care, since denoising which involves smoothing of the noisy signal (e.g., using low-pass flter) may cause the loss of fine details. In the context of denoising, the success of techniques based on the wavelet theory is ensured by the ability of decorrelation (separation of noise and useful signal) of the different discrete wavelet transforms. Because the signal is contained in a small number of coefficients of such a transform, all other coefficients essentially contain noise.

By filtering these coefficients, most of the noise is eliminated. Thus, each method of image denoising based on the use of wavelets follows the classic method, in three steps: computing a discrete wavelet transform of the image to be denoised, filtering in the wavelet domain and the computation of the corresponding inverse wavelet transform. Throughout recent years, many wavelet transforms (WT) have been used to operate denoising. The first one was the discrete wavelet transform. It has three main disadvantages: lack of shift invariance, lack of symmetry of the mother wavelet and poor directional selectivity. These disadvantages can be diminished using a complex wavelet transform. More than 20 years ago, Grossman and Morlet developed the continuous wavelet transform. A revival of interest in later years has occurred in both signal processing and statistics for the use of complex wavelets, and complex analytic wavelets, particularly in. It may be linked to the development of complex-valued discrete wavelet filters and the clever dual filter bank. The complex WT has been shown to provide a powerful tool in signal and image analysis.

II. DUAL TREE DISCRETE WAVELET TRANSFORM (DT-DWT)

The classical discrete wavelet transform (DWT) provides a means of implementing a multiscale analysis, based on a critically sampled filter bank with perfect reconstruction. However, questions arise regarding the good qualities or properties of the wavelets and the results obtained using these tools, the standard DWT suffers from the following problems described as below:

1. *Shift sensitivity:* It has been observed that DWT is seriously disadvantaged by the shift sensitivity that arises from down samples in the DWT implementation.
2. *Poor directionality:* an m-dimension transform (m>1) suffers poor directionality when the transform coefficients reveal only a few feature in the spatial domain.
3. *Absence of phase information:* filtering the image with DWT increases its size and adds phase distortions; human visual system is sensitive to phase distortion. Such DWT implementations cannot provide the local phase information.

In other applications, and for certain types of images, it is necessary to think of other, more complex wavelets, who gives a good way, because the complex wavelets filters which can be made to suppress negative frequency components. The complex wavelet transform has improved shift-invariance and directional selectivity. This implementation uses consists in analyzing the signal by two different DWT trees, with filters chosen so that at the end, the signal returns with the approximate decomposition by an analytical wavelet. The dual-tree structure has an extension of conjugate filtering in 2-D case. Because of the existence of two trees the second noise coefficients moments from such decomposition can be precisely characterized. The DT-DWT ensures filtering of the results without distortion and with a good ability for the localization function and the perfect reconstruction of signal. In the noise study, as with any redundant frame analysis, when a stationary noise, even if white, is subject to a dual decomposition tree, statistical dependencies appear between coefficients, because of the existence of two trees, it appears that the second noise coefficients moments from such decomposition can be precisely characterized. We observe a de-correlation between primal and dual coefficients located at the same spatial position and an inter-scale correlation, which allows us to choose between several estimators, taking this phenomenon into account. If we consider an image degraded by centered, additive Gaussian noise with a spectral density, the decomposition coefficients are also affected by that same noise as part of the linearity property. With this advantage we can choose an appropriate estimator for de-noising. In the case of DT-DWT the mathematical expression for a signal observed at point whose coordinates (x,y) in the image is modeled as follows:

$$g(x,y) = f(x,y) + \varepsilon(x,y)$$

With g(x,y) , f(x,y) and ε(x,y) are respectively the noise coefficient, the original coefficient, and the Gaussian independent noise. After applying the DT-DWT on g(x,y) we obtain:

$$g_\eta(x,y) = f_\eta(x,y) + \varepsilon_\eta(x,y)$$

Where, $g_\eta(x,y)$, ,$f_\eta(x,y)$ and $\varepsilon_\eta.(x,y)$ denote $(x,y)^{th}$ wavelet coefficient at level of a particular detail subband of the DT-DWT of g, f, and ε , respectively and ŋ (ŋ= 1,2…,J).

The dual-tree complex DWT of a signal x is implemented using two critically-sampled DWTs in parallel on the same data. The transform is 2-times expansive because for an N-point signal it gives 2N DWT coefficients. If the filters in the upper and lower DWTs are the same, then no advantage is gained. However, if the filters are designed is a specific way, then the sub band signals of the upper DWT can be interpreted as the real part of a complex wavelet transform, and sub band signals of the lower DWT can be interpreted as the imaginary part.

Equivalently, for specially designed sets of filters, the wavelet associated with the upper DWT can be an approximate Hilbert transform of the wavelet associated with the lower DWT. When designed in this way, the dual-tree complex DWT is nearly shift-invariant, in contrast with the critically-sampled DWT. Moreover, the dual-tree complex DWT can be used to implement 2D wavelet transforms where each wavelet is oriented, which is especially useful for image processing. The dual-tree DWT outperforms the critically sampled DWT for applications like image denoising and enhancement.

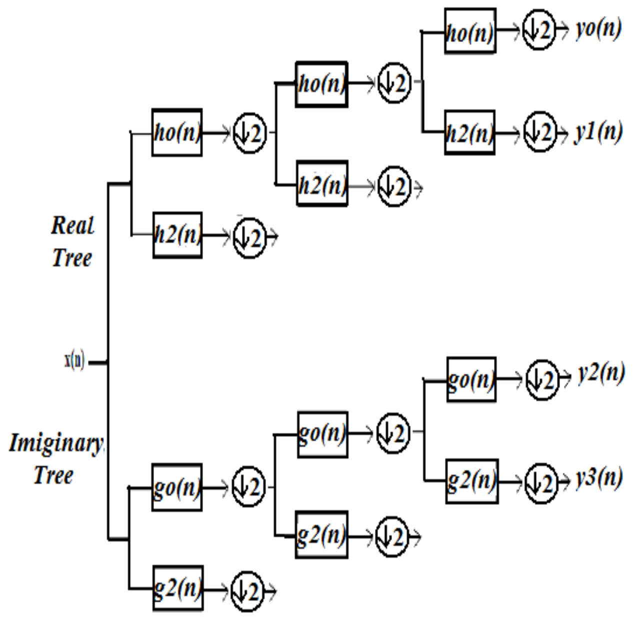

Figure 1: Implementation of Dual-Tree Discrete Wavelet Transform.

III. Wiener Filter

This filter was proposed by Norbert Wiener during the 1940s and published in 1949. The discrete-time equivalent of Wiener's work was derived independently by Andrey Kolmogorov and published in 1941. Hence the theory is often called the Wiener–Kolmogorov filtering theory (cf. Kriging). The Wiener filter was the first statistically designed filter to be proposed and subsequently gave rise to many others including the Kalman filter. The Wiener filter is the MSE-optimal stationary linear filter for images degraded by additive noise and blurring. Wiener filters are usually applied in the frequency domain..

IV. Parameters under consideration

1) PSNR: PSNR stands for the peak signal to noise ratio. It is a term used to calculate the ratio between the maximum possible power of a signal and the power of corrupting noise that affects the fidelity of its representation It is most commonly used as a measure of quality of reconstruction in image compression etc.. Because many signals have a very wide dynamic range, (ratio between the largest and smallest possible values of a changeable quantity) the PSNR is usually expressed in terms of the logarithmic decibel scale.

The mathematical representation of the **PSNR** is as follows:

$$PSNR = 20\log_{10}\left(\frac{MAX_f}{\sqrt{MSE}}\right)$$

MSE indicates average error of the pixels throughout the image. In our work, a definition of a higher MSE does not indicate that the denoised image suffers more errors instead it refers to a greater difference between the original and denoised image. This means that there is a significant speckle reduction.

where the ***MSE*** *(Mean Square Error)* is:

$$MSE = \frac{1}{mn}\sum_{0}^{m-1}\sum_{0}^{n-1}\|f(i,j) - g(i,j)\|^2$$

f represents the matrix data of our original image
g represents the matrix data of our degraded image in question
m represents the numbers of rows of pixels of the images and i represents the index of that row

n represents the number of columns of pixels of the image and **j** represents the index of that column.

$\mathbf{MAX_f}$ is the maximum signal value that exists in our original "known to be good" image. [1][4][11][22]

2) ***Structural Similarity (SSIM) index and SSIM Map:*** The Structural Similarity (SSIM) index is a novel method for measuring the similarity between two images. The SSIM index can be viewed as a quality measure of one of the images being compared provided the other image is regarded as of perfect quality. The Structural Similarity Index (SSIM) is a perceptual metric that quantifies image quality degradation caused by processing such as data compression or by losses in data transmission. It is a full reference metric that requires two images from the same image capture— a reference image and a processed image. SSIM is best known in the video industry, but has strong applications for still photography. Any image may be used. Unlike PSNR (Peak Signal-to-Noise Ratio), SSIM is based on visible structures in the image. Although PSNR is no longer regarded as a reliable indicator of image quality degradation it is available as an alternative measurement in the SSIM module. The SSIM is designed to improve on traditional metrics like PSNR and MSE, which have proved to be inconsistent with human eye perception.

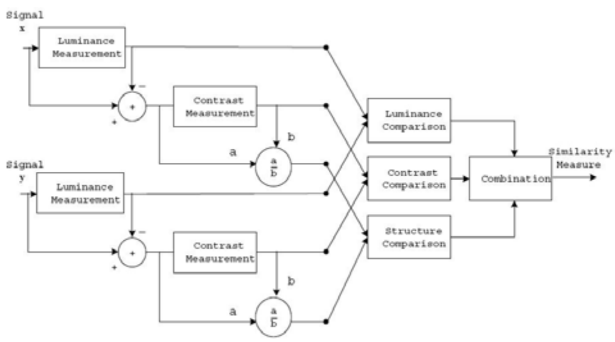

Fig 2: Structural Similarity (SSIM) Measurement System.

V. Steps involved in the proposed algorithm.

Steps involved in the proposed system are:

1. The original medical image is read.
2. The read image is converted to grey image.
3. Transform the grey image into double data.
4. Grey image is passed through AWGN channel.
5. The noise is removed using dual tree discrete wavelet transform (DT-DWT).
6. After denoising using DT-DWT the image is further enhances by filtering out noise using wiener filter.
7. The outputs are analyzed on the basis of Peak Signal to Noise Ratio (PSNR), Structural Similarity Index (SSIM) and SSIM Index Map.

The simulation diagram is given below

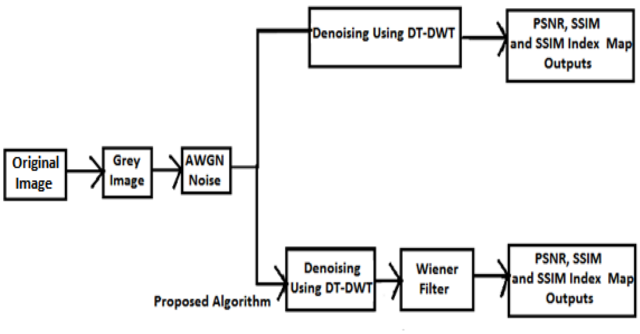

Figure 3: Simulation Diagram.

VI. SIMULATION RESULTS

The proposed algorithm is analyzed for three outputs Peak Signal to Noise Ratio (PSNR), Structural Similarity Index (SSIM) and SSIM Index Map.

Figure 4: Original Medical Image.

Figure 5: Grey Image.

Figure 6: Image with AWGN noise.

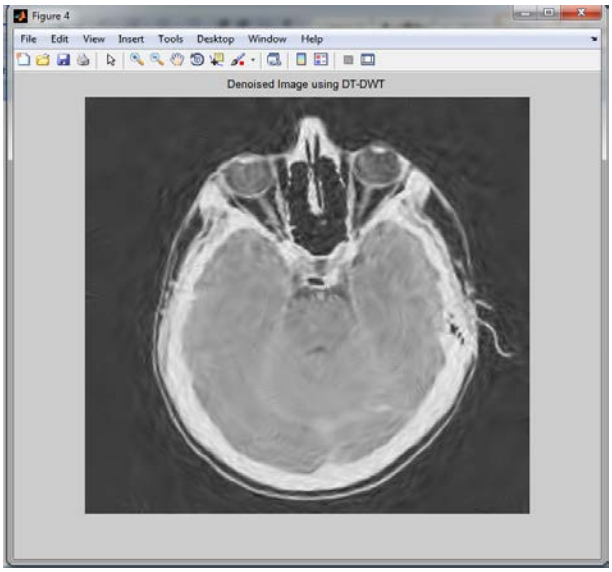

Figure 7: Denoised image with DT-DWT.

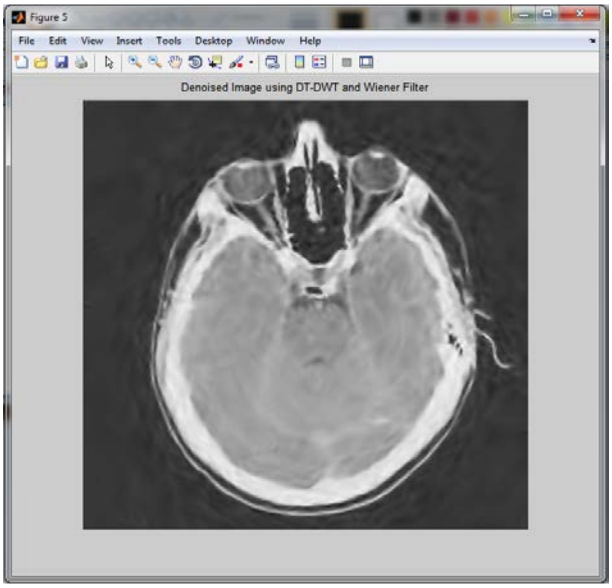

Figure 8: Denoised image with DT-DWT and Wiener filter.

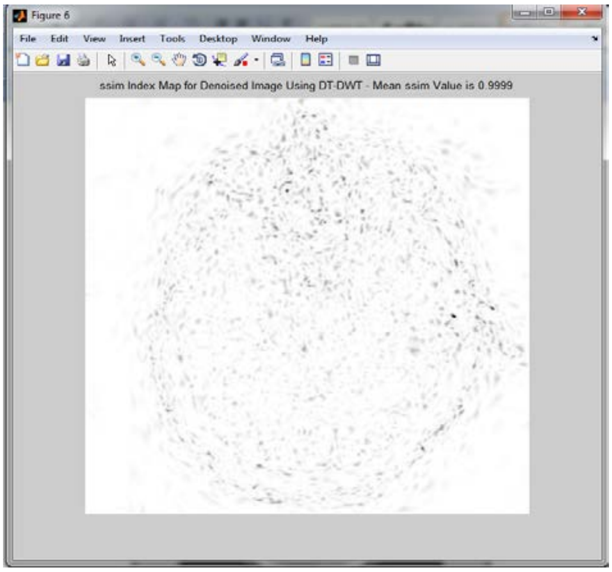

Figure 9: SSIM Index Map using DT-DWT.

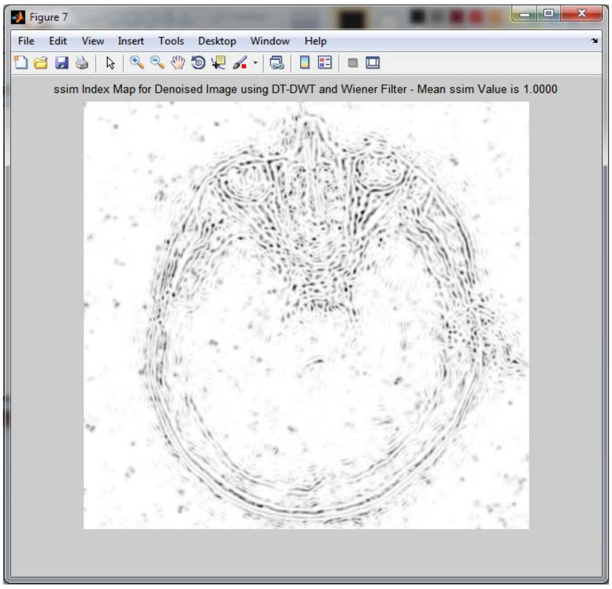

Figure 10: SSIM Index Map using DT-DWT and Wiener filter.

The figures from 4 to 10 shows different output images obtained during simulation of the system. Figure 9 and figure 10 shows that the SSIM index map for proposed system is better than the traditional system.

Table 1: Analysis of parameters using different algorithms.

PARAMETERS	WITH DTDWT	WITH PROPOSED ALGORITHM
PSNR	75.3548	75.7541
SSIM	0.9999	1.0

The table 1 shows the PSNR and SSIM outputs obtained from different algorithms. From the table it is clear that PSNR and SSIM parameters of proposed system are improved in comparison to that with DT-DWT.

VII. CONCLUSION

The objective of this work is to improve the Peak Signal to Noise Ratio (PSNR), Structural Similarity Index (SSIM) and SSIM Index Map of medical image by denoising the image using Dual Tree Discrete Wavelet Transform (DT-DWT) and Wiener filter. In the proposed system outputs are derived for denoising system with DT-DWT alone and with DT-DWT and Wiener filter both. The parameters PSNR and SSIM are compared for an image affected by white Gaussian noise, SSIM index map is also generated for the systems. The results show that the proposed system performs better than the existing system and hence it can be concluded that the system performance is enhanced in terms of parameters under consideration.

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