On Nano (1,2)* α - Generalized Closed Sets in Nano Bitopological Spaces

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Abstract: The purpose of this paper is to define and study a new class of sets called Nano (1,2)* α - generalized closed set in Nano bitopological spaces. Some of its properties are investigated.

Keywords: Nano (1,2)* α- generalized closed sets, Nano (1,2)* α-closure, Nano (1,2)*α-interior.

I. INTRODUCTION

In 1965 O.Njastad [8] introduced and defined an α - open and closed set. The concepts of Nano topology was introduced by Lellis Thivagar[5]. K.Bhuvaneswari et al.,[1] introduced and studied the Nano α - generalized closed sets in Nano topological spaces. In 1963, J.C.Kelly[4] introduced the study of bitopological spaces. In 1990, M.Jelic[3] introduced the concepts of α - open sets in bitopological space. O.A.EI-Tantaway et al., [9] introduced α - generalized closed sets in bitopological spaces. In this paper a new set called Nano (1,2)* α - generalized closed sets in Nano bitopological space introduced and studied.

II. PRELIMINARIES

Definition:2.1 [8] A subset A of a space (X,τ) is called α - open if $A \subseteq Int(cl(Int(A)))$.

A subset A of X is called α - closed if $cl(Int(cl(A))) \subseteq A$.

Definition:2.2 [7] A subset A of (X,τ) is called α - generalized closed (briefly αg -closed) if αCl(A)$\subseteq$U whenever A$\subseteq$U and U is open in (X,τ).

Definition:2.3 [5] Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as the indiscernibility relation. Then U is divided into disjoint equivalence classes. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U,R) is said to be the approximation space. Let X$\subseteq$U.

i. The lower approximation on X with respect to R is the set of all objects which can be for certain classified as X with respect to R and is denoted by $L_R(X)$. That is, $L_R(X) = U\{R(X) : R(X) \subseteq X, x \in U\}$, where $R(X)$ denotes the equivalence class determined by $x \in U$.

ii. The upper approximation of X with respect to R is the of all objects which can be possibly classified as X with respect to R and is denoted by $U_R(X)$.
That is, $U_R(X) = U\{R(X) : R(X) \cap X \neq \phi, x \in U\}$

iii. The boundary region of X with respect to R is the set of all objects which can be classified neither as X nor as not X with respect to R and it is denoted by $B_R(X)$.
That is, $B_R(X) = U_R(X) - L_R(X)$.

Definition:2.4 [5] Let U be the universe, R be an equivalence relation on U and $\tau_R(X) = \{U, \phi, L_R(X), U_R(X), B_R(X)\}$. Where X$\subseteq$U. $\tau_R(X)$ satisfies the following axioms:

i) U and $\phi \in \tau_R(X)$.
ii) The union of the elements of any sub collection of $\tau_R(X)$ is in $\tau_R(X)$.
iii) The intersection of the elements of any finite sub collection of $\tau_R(X)$ is in $\tau_R(X)$.

That is, $\tau_R(X)$ is a topology on U called the Nano topology on U with respect to X. We call (U, $\tau_R(X)$) as the Nano topological space. The elements of $\tau_R(X)$ are called as Nano open sets and [$\tau_R(X)$]^c is called as Nano closed sets.

Definition:2.5 [6] Let (U, $\tau_R(x)$) be a Nano topological space and A $\subseteq$ U. Then A is said to be a Nano α-open if A $\subseteq$ NInt [Ncl (NInt(A))]

The compliment of a Nano α-open set of a space X is called Nano α - closed set in X.

Definition:2.6 [1] Let (U, $\tau_R(x)$) be a Nano topological space and A $\subseteq$ U. Then A is said to be Nano α - generalized closed set if Nαcl(A) $\subseteq$ V whenever A $\subseteq$ V and V is Nano open in U.

Definition:2.7 [3] A subset A of a bitopological space (X, $\tau_{1,2}$) is called

i) (1,2)* -α-open if A $\subseteq$ $\tau_{1,2}Int(\tau_{1,2}cl(\tau_{1,2}Int(A)))$
ii) (1,2)*-α-closed if A $\subseteq$ $\tau_{1,2}cl(\tau_{1,2}Int(\tau_{1,2}cl(A)))$

Definition:2.8 [9] A subset of a bitopological space (X, $\tau_{1,2}$) is called

i) (1,2)* α-generalized closed (briefly (1,2)*αg-closed) if $\tau_{1,2}\alpha Cl(A) \subseteq U$ whenever A $\subseteq$ U and U is (1,2)* open in X.
ii) (1,2)* α - generalized open (briefly (1,2)*αg-open) if X-A is (1,2)* α - generalized closed.

Definition:2.9 [2] Let U be the universe, R be an equivalence relation on U and $\tau_{R_{1,2}}(X) = \cup\{\tau_{R_1}(X), \tau_{R_2}(X)\}$ where $\tau_R(X) = \{U, \phi, L_R(X), U_R(X), B_R(X)\}$ and $X \subseteq U$. Then $(U, \tau_{R_{1,2}}(X))$ is called the nano bitopological space. Elements of the nano bitopology are known as nano (1, 2)* open sets in U. Elements of $[\tau_{R_{1,2}}(X)]^c$ are called nano (1, 2)* closed sets in $\tau_{R_{1,2}}(X)$.

Definition:2.10 [2] If $(U, \tau_{R_{1,2}}(X))$ is a Nano bitopological space with respect to X where $X \subseteq U$ and if $A \subseteq U$, then

(i) The Nano (1,2)* closure of A is defined as the intersection of all Nano (1,2)* closed sets containing A and it is denoted by $N\tau_{1,2}cl(A)$. $N\tau_{1,2}cl(A)$ is the smallest Nano (1,2)* closed set containing A.

(ii) The Nano (1,2)* interior of A is defined as the union of all Nano (1,2)* open subsets of A contained in A and it is denoted by $N\tau_{1,2}Int(A)$. $N\tau_{1,2}Int(A)$ is the largest Nano (1,2)* open subset of A.

III. NANO (1,2)* α - GENERALIZED CLOSED SETS IN NANO BITOPOLOGICAL SPACE

Definition:3.1 Let $(U, \tau_{R_{1,2}}(X))$ be a Nano bitopological space and $A \subseteq U$. Then A is said to be a Nano (1,2)* α-open if $A \subseteq N\tau_{1,2}Int[N\tau_{1,2}cl(N\tau_{1,2}Int(A))]$.

The compliment of a Nano (1,2)* α - open set of a space X is called Nano (1,2)* α - closed set in X.

Definition:3.2 If $(U, \tau_{R_{1,2}}(X))$ is a Nano bitopological space with respect X where X $\subseteq$ U and if A $\subseteq$ U, then

The Nano (1,2)* α-closure of a set A is defined as the intersection of all Nano (1,2)* α - closed sets containing A and it is denoted by $N\tau_{1,2}\alpha cl(A)$. $N\tau_{1,2}\alpha cl(A)$ is the smallest Nano (1,2)* α - closed set containing A.

The Nano (1,2)* α-interior of a set A is defined as the union of all Nano (1,2)* α - open subsets contained in A and it is denoted by $N\tau_{1,2}\alpha Int(A)$. $N\tau_{1,2}\alpha Int(A)$ is the largest Nano (1,2)* α-open subset of A.

Definition:3.3 A subset A of $(U, \tau_{R_{1,2}}(X))$ is called a Nano (1,2)* α- generalized closed set if $N\tau_{1,2}\alpha cl(A) \subseteq V$ whenever A $\subseteq$ V and V is Nano (1,2)* open in V.

Theorem:3.4 Let $(U, \tau_{R_{1,2}}(X))$ be a Nano bitopological space. If a subset A of a Nano bitopological space $(U, \tau_{R_{1,2}}(X))$ is Nano (1,2)* closed set, then A is a Nano (1,2)* α - generalized closed set.

Proof: Let A be a Nano (1,2)* closed set of U and A $\subseteq$ V, V is Nano (1,2)* α-open in U. Since every Nano (1,2)* open set is Nano (1,2)* α open. Therefore V is Nano (1,2)* α-open in U. Here A is Nano (1,2)* closed, $N\tau_{1,2}cl(A) = V$, A $\subseteq$ V, this implies $N\tau_{1,2}\alpha cl(A) \subseteq V$.

Also $N\tau_{1,2}\alpha cl(A) \subseteq N\tau_{1,2}cl(A)$ which implies $N\tau_{1,2}\alpha cl(A) \subseteq V$ whenever A$\subseteq$V and V is Nano (1,2)*α- open in U. Therefore A is Nano (1,2)* α-generalized closed set.

Remark:3.5 The converse of the above Theorem 3.4 is not true. Which has been seen from the following example.

Let U= {a,b,c,d} with

U/R$_1$= {{a},{d},{b,c}} and X$_1$ = {b,c} Then $\tau_{R_1}(x)$ ={U, ϕ ,{b,c}}.

U/R$_2$ ={{a},{c},{b,d}} and X$_2$= {b,d} Then $\tau_{R_2}(x)$ ={U, ϕ ,{a},{c},{b,d}}

$\tau_{R_{1,2}}(x)$ ={U, ϕ ,{b,c},{b,d}} are Nano $(1,2)*$ open sets.

$[\tau_{R_{1,2}}(x)]^c$ = {U, ϕ ,{a,d},{a,c}} are Nano (1,2)* closed sets.

Then{U, ϕ ,{a},{a,b},{a,c},{a,d},{c,d},{a,b,c}, {a,b,d},{a,c,d},{b,c,d}} are Nano (1,2)*α- generalized closed sets but are not Nano (1,2)* closed sets.

Theorem:3.6 Let $(U,\tau_{R_{1,2}}(X))$ be a Nano bitopological space. If a subset A of a Nano bitopological space $(U,\tau_{R_{1,2}}(X))$ is Nano (1,2)* generalized closed set, then A is a Nano (1,2)* α - generalized closed set.

Proof: Let A be a Nano (1,2)* generalized closed set. Then $N\tau_{1,2}cl(A) \subseteq V$ whenever A$\subseteq$V, V is Nano (1,2)* open in U. Since every Nano (1,2)* open set is Nano(1,2)* α - open set.

Therefore V is Nano (1,2)* α-open in U. Also $N\tau_{1,2}\alpha cl(A) \subseteq N\tau_{1,2}cl(A)$, this implies $N\tau_{1,2}\alpha cl(A) \subseteq V$ whenever A$\subseteq$V and V is Nano (1,2)* α - open in U. Therefore A is a Nano (1,2)* α-generalized closed set.

Theorem:3.7 The union of two Nano (1,2)* α-generalized closed set in $(U,\tau_{R_{1,2}}(X))$ are also Nano (1,2)* α - generalized closed set in $(U,\tau_{R_{1,2}}(X))$.

Proof: Let A and B be two Nano (1,2)* α - generalized closed set in $(U,\tau_{R_{1,2}}(X))$. Let V be a Nano (1,2)* α -open set in U. such that A$\subseteq$V and B$\subseteq$V. Then we have $A\cup B \subseteq V$, Since A and B are Nano (1,2)* α -generalized closed set in $(U,\tau_{R_{1,2}}(X))$. $N\tau_{1,2}\alpha cl(A) \subseteq V$ and $N\tau_{1,2}\alpha cl(B) \subseteq V$ .Now, $N\tau_{1,2}\alpha cl(A\cup B) \subset N\tau_{1,2}\alpha cl(A) \cup N\tau_{1,2}\alpha cl(B)$ this implies $N\tau_{1,2}\alpha cl(A\cup B) \subseteq$V whenever $A\cup B \subseteq V$ and V is Nano (1,2)* α - open in ($U,\tau_{R_{1,2}}(X)$). Thus A$\cup$B is a Nano (1,2)* α-generalized closed set in $(U,\tau_{R_{1,2}}(X))$.

Remark:3.8

The intersection of two Nano (1,2)* α - generalized closed set in $(U,\tau_{R_{1,2}}(X))$ are need not to be Nano (1,2)* α - generalized closed set in $(U,\tau_{R_{1,2}}(X))$. Let $U=\{a,b,c,d\}$

$U/R_1 = \{\{a\},\{d\},\{b,c\}\}$ and $X_1 = \{b,c\}$ Then $\tau_{R_1}(X) = \{U,\phi,\{b,c\}\}$

$U/R_2 = \{\{a\},\{c\},\{b,d\}\}$ and $X_2 = \{b,d\}$ Then $\tau_{R_2}(X) = \{U,\phi,\{b,d\}\}$

$\tau_{R_{1,2}}(X) = \{U,\phi,\{b,c\},\{b,d\}\}$ are Nano (1,2)* open sets.

$[\tau_{R_{1,2}}(X)]^c = \{U,\phi,\{a,d\},\{a,c\}\}$ are Nano (1,2)* closed sets.

Nano (1,2)* α - generalized closed sets

$\{U,\phi,\{a\},\{a,b\},\{a,c\},\{a,d\},\{c,d\},\{a,b,c\},\{a,b,d\},\{a,c,d$

Hence The intersection of two Nano (1,2) α-generalized closed set in $(U,\tau_{R_{1,2}}(X))$ are need not to be Nano (1,2)* α- generalized closed set in $(U,\tau_{R_{1,2}}(X))$.

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