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Soft Fixed-Point Theorems for Integral Type Mappings through Rational Expression

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Abstract - In the present research paper,some soft fixed-pointtheorems are established for integral type mappings for rational expressions. These results are proved by the help of basic concepts of fixed-pointtheory. To obtain the results altering distance functions are used. Obtained results are generalized form of well-knownresults in complete metric spaces.

I. INTRODUCTION & PRELIMINARIES

A new category of contractive fixed point problem was introduced by M. S. Khan,M. Swalech and S. Sessa [10]. In this work, they introduced the concept of altering distance function which is a control function that alters distance between two points in a metric space.

In the year 1999, Molodtsov [14] initiated a novel concept of soft sets theory as a new mathematical tool for dealing with uncertainties. Detail about soft sets, soft points, soft fixed point theorems can be seen in [4-8, 11-13],The present work delt with some soft fixed point results for rational expressions using altering distance function for softmetric space, which is motivated by Molodtsov[14] , Khan M.S. et.al [10] . and Binciri [3]

Definition1.A: Let X be an initial universe set and E be a set of parameters. A pair (F,E) is called a soft set over X if and only if X is a mapping from E into the set of all subsets of the set $X, i.e. F: E \to P(X)$,where $P(X)$ is the power set of X.

Definition 1B: Let$\mathfrak{R}$ be the set of real numbers and $B(\mathfrak{R})$ be the collection of all nonempty bounded subsets of $\mathfrak{R}$ and Etaken as a set of parameters. Then a mapping $F: E \to B(\mathfrak{R})$ is called a soft real set. It is denoted by (F,E). If specifically (F,E) is a singleton soft set, then identifying (F,E) with the corresponding soft element, it will be called a soft real number and denoted $\tilde{r}, \tilde{s}, \tilde{t}$ etc.

$\overline{0}, \overline{1}$are the soft real numbers where $\overline{0}(e) = 0, \overline{1}(e) = 1$ for all $e \in E$,respectively.

Definition 1C: A soft set over X is said to be a soft point if there is exactly one $e \in E$, such that $P(e) = \{x\}$ for some $x \in X$ and $P(e') = \phi, \forall e' \in E \backslash \{e\}$. It will be denoted by $\tilde{x}_e$.

Definition 1D: Two soft points $\tilde{x}_e, \tilde{y}_e$ are said to be equal if $e = e'$ and $P(e) = P(e')$ i.e. $x = y$. Thus $\tilde{x}_e \neq \tilde{y}_e \Leftrightarrow x \neq y$ or $e \neq e'$.

Definition 1 E: A mapping $\tilde{d}: SP(\tilde{X}) \times SP(\tilde{X}) \to \mathbb{R}(E)^*$, is said tobe a soft metric on the soft set $\tilde{X}$ if $d$satisfies the following conditions:

(M1) $\tilde{d}(\tilde{x}_{e_1}, \tilde{y}_{e_2}) \cong \overline{0}$ for all $\tilde{x}_{e_1}, \tilde{y}_{e_2} \tilde{\in} \tilde{X}$,

(M2) $\tilde{d}(\tilde{x}_{e_1}, \tilde{y}_{e_2}) = \overline{0}$ if and only if$\tilde{x}_{e_1} = \tilde{y}_{e_2}$,

(M3) $\tilde{d}(\tilde{x}_{e_1}, \tilde{y}_{e_2}) = \tilde{d}(\tilde{y}_{e_2}, \tilde{x}_{e_1})$ for all $\tilde{x}_{e_1}, \tilde{y}_{e_2} \tilde{\in} \tilde{X}$,

(M4) $\tilde{d}(\tilde{x}_{e_1}, \tilde{z}_{e_3}) \tilde{\leq} \tilde{d}(\tilde{x}_{e_1}, \tilde{y}_{e_2}) + \tilde{d}(\tilde{y}_{e_2}, \tilde{z}_{e_3})$ for all $\tilde{x}_{e_1}, \tilde{y}_{e_2}, \tilde{z}_{e_3} \tilde{\in} \tilde{X}$.

The soft set $\tilde{X}$ with a soft metric $\tilde{d}$ on $\tilde{X}$ is called a soft metric space and denoted by $(\tilde{X}, \tilde{d}, E)$.

Definition 1F(Soft Complete Metric Space): A soft metric space $(\tilde{X}, \tilde{d}, E)$is called complete, if every Cauchy Sequence in$\tilde{X}$converges to some point of $\tilde{X}$.

Definition 1 G: Let$(\tilde{X}, \tilde{d}, E)$be a soft metric space. A function$(f,\varphi) : (\tilde{X}, \tilde{d}, E) \to (\tilde{X}, \tilde{d}, E)$is called a soft contractive mapping if there exist a soft real number $\alpha \in R, 0 \leq \alpha < 1$such that forevery point$\tilde{x}_\lambda, \tilde{y}_\mu \in SP(X)$we have

$$\tilde{d}\left((f,\varphi)(\tilde{x}_\lambda), (f,\varphi)(\tilde{y}_\mu)\right) \leq \alpha \tilde{d}(\tilde{x}_\lambda, \tilde{y}_\mu)$$

Definition 1H[9]: The function $\psi : [0,\infty) \to [0,\infty)$ is called an altering distance function if the following properties are satisfied:

(i) ψ is continuous and non-decreasing,

(ii) $\psi(t) = 0$ if and only if $t = 0$.

II. MAIN RESULTS

Theorem 2.1: Let$(\tilde{X}, \tilde{d}, E)$be a soft complete metric space. Suppose the soft mapping$(f,\varphi) : (\tilde{X}, \tilde{d}, E) \to (\tilde{X}, \tilde{d}, E)$satisfies the soft contractive condition:

$\psi\left[\tilde{d}\left((f,\varphi)(\tilde{x}_\lambda),(f,\varphi)(\tilde{y}_\mu)\right)\right] \leq \psi[\widetilde{\mathcal{M}}(\tilde{x}_\lambda,\tilde{y}_\mu)] - \varphi[\widetilde{\mathcal{M}}(\tilde{x}_\lambda,\tilde{y}_\mu)]$(2.1.1)

For each $\tilde{x}_\lambda,\tilde{y}_\mu \in \tilde{X}, \tilde{x}_\lambda \neq \tilde{y}_\mu$,where ψ,φ are altering distance functions, and

$$\widetilde{\mathcal{M}}(\tilde{x}_\lambda,\tilde{y}_\mu) = \alpha\int_0^{\left\{\frac{\tilde{d}^3(\tilde{x}_\lambda,(f,\varphi)(\tilde{x}_\lambda))+\tilde{d}^3(\tilde{y}_\mu,(f,\varphi)(\tilde{y}_\mu))}{1+\tilde{d}^2(\tilde{x}_\lambda,(f,\varphi)(\tilde{x}_\lambda))+\tilde{d}^2(\tilde{y}_\mu,(f,\varphi)(\tilde{y}_\mu))}\right\}}\xi(t)dt+\gamma\int_0^{\{\tilde{d}(\tilde{x}_\lambda,\tilde{y}_\mu)\}}\xi(t)dt$$

Where $\alpha,\gamma > 0$and$2\alpha+\gamma < 1$is a soft constant. Then (f,φ)has a unique soft fixed point in$\tilde{X}$.

Proof: Let $\tilde{x}_\lambda^0$be any soft point in$SP(X)$.

Set$\tilde{x}_{\lambda_1}^1 = (f,\varphi)(\tilde{x}_\lambda^0) = \left(f(\tilde{x}_\lambda^0)\right)_{\varphi(\lambda)}$

$$\tilde{x}_{\lambda_{n+1}}^{n+1} = (f,\varphi)(\tilde{x}_{\lambda_n}^n) = \left(f^{n+1}(\tilde{x}_\lambda^0)\right)_{\varphi^{n+1}(\lambda)}, --$$

Now consider,

$$\widetilde{\mathcal{M}}(\tilde{x}_{\lambda_{n-1}}^{n-1},\tilde{x}_{\lambda_n}^n) = \alpha\int_0^{\left\{\frac{\tilde{d}^3\left(\tilde{x}_{\lambda_{n-1}}^{n-1},(f,\varphi)(\tilde{x}_{\lambda_{n-1}}^{n-1})\right)+\tilde{d}^3\left(\tilde{x}_{\lambda_n}^n,(f,\varphi)(\tilde{x}_{\lambda_n}^n)\right)}{1+\tilde{d}^2\left(\tilde{x}_{\lambda_{n-1}}^{n-1},(f,\varphi)(\tilde{x}_{\lambda_{n-1}}^{n-1})\right)+\tilde{d}^2\left(\tilde{x}_{\lambda_n}^n,(f,\varphi)(\tilde{x}_{\lambda_n}^n)\right)}\right\}}\xi(t)dt$$

$$+\gamma\int_0^{\left\{\tilde{d}\left(\tilde{x}_{\lambda_{n-1}}^{n-1},\tilde{x}_{\lambda_n}^n\right)\right\}}\xi(t)dt$$

$$=\alpha\int_0^{\left\{\frac{\tilde{d}^3(\tilde{x}_{\lambda_{n-1}}^{n-1},\tilde{x}_{\lambda_n}^n)+\tilde{d}^3(\tilde{x}_{\lambda_n}^n,\tilde{x}_{\lambda_{n+1}}^{n+1})}{1+\tilde{d}^2(\tilde{x}_{\lambda_{n-1}}^{n-1},\tilde{x}_{\lambda_n}^n)+\tilde{d}^2(\tilde{x}_{\lambda_n}^n,\tilde{x}_{\lambda_{n+1}}^{n+1})}\right\}}\xi(t)dt+\gamma\int_0^{\left\{\tilde{d}\left(\tilde{x}_{\lambda_{n-1}}^{n-1},\tilde{x}_{\lambda_n}^n\right)\right\}}\xi(t)dt$$

$$\leq(\alpha+\gamma)\int_0^{\tilde{d}\left(\tilde{x}_{\lambda_{n-1}}^{n-1},\tilde{x}_{\lambda_n}^n\right)}\xi(t)dt+(\alpha)\int_0^{\tilde{d}\left(\tilde{x}_{\lambda_n}^n,\tilde{x}_{\lambda_{n+1}}^{n+1}\right)}\xi(t)dt$$

So by the definition we can have

$$\psi[\tilde{d}(\tilde{x}_{\lambda_n}^n,\tilde{x}_{\lambda_{n+1}}^{n+1})] = \psi\left[\tilde{d}\left((f,\varphi)(\tilde{x}_{\lambda_{n-1}}^{n-1}),(f,\varphi)(\tilde{x}_{\lambda_n}^n)\right)\right]$$

$$\leq\psi[\widetilde{\mathcal{M}}(\tilde{x}_{\lambda_{n-1}}^{n-1},\tilde{x}_{\lambda_n}^n)]-\varphi[\widetilde{\mathcal{M}}(\tilde{x}_{\lambda_{n-1}}^{n-1},\tilde{x}_{\lambda_n}^n)]$$

$$\leq\psi[(\alpha+\gamma)\int_0^{\tilde{d}\left(\tilde{x}_{\lambda_{n-1}}^{n-1},\tilde{x}_{\lambda_n}^n\right)}\xi(t)dt+(\alpha)\int_0^{\tilde{d}\left(\tilde{x}_{\lambda_n}^n,\tilde{x}_{\lambda_{n+1}}^{n+1}\right)}\xi(t)dt]$$

$$-\varphi\left[\widetilde{\mathcal{M}}(\tilde{x}_{\lambda_{n-1}}^{n-1},\tilde{x}_{\lambda_n}^n)\right]$$

$$\psi[\tilde{d}(\tilde{x}_{\lambda_n}^n,\tilde{x}_{\lambda_{n+1}}^{n+1})]\leq\psi[(\alpha+\gamma)\int_0^{\tilde{d}\left(\tilde{x}_{\lambda_{n-1}}^{n-1},\tilde{x}_{\lambda_n}^n\right)}\xi(t)dt+(\alpha)\int_0^{\tilde{d}\left(\tilde{x}_{\lambda_n}^n,\tilde{x}_{\lambda_{n+1}}^{n+1}\right)}\xi(t)dt]$$

Since $\psi$is non-decreasing, we have

$$\int_0^{\tilde{d}\left(\tilde{x}_{\lambda_n}^n,\tilde{x}_{\lambda_{n+1}}^{n+1}\right)}\xi(t)dt\leq(\alpha+\gamma)\int_0^{\tilde{d}\left(\tilde{x}_{\lambda_{n-1}}^{n-1},\tilde{x}_{\lambda_n}^n\right)}\xi(t)dt+(\alpha)\int_0^{\tilde{d}\left(\tilde{x}_{\lambda_n}^n,\tilde{x}_{\lambda_{n+1}}^{n+1}\right)}\xi(t)dt$$

$$\int_0^{\tilde{d}\left(\tilde{x}_{\lambda_n}^n,\tilde{x}_{\lambda_{n+1}}^{n+1}\right)}\xi(t)dt\leq\frac{(\alpha+\gamma)}{(1-\alpha)}\int_0^{\tilde{d}\left(\tilde{x}_{\lambda_{n-1}}^{n-1},\tilde{x}_{\lambda_n}^n\right)}\xi(t)dt$$

$\int_0^{\tilde{d}\left(\tilde{x}_{\lambda_n}^n,\tilde{x}_{\lambda_{n+1}}^{n+1}\right)}\xi(t)dt\leq s^n\int_0^{\tilde{d}\left(\tilde{x}_{\lambda_0}^0,\tilde{x}_{\lambda_1}^1\right)}\xi(t)dt$, Where $s=\frac{\alpha+\gamma}{1-\alpha}$

Taking$n\to\infty$,we have

$\lim_{n\to\infty}\tilde{d}(\tilde{x}_{\lambda_n}^n,\tilde{x}_{\lambda_{n+1}}^{n+1}) = 0$(2.1.2)

Now, we will show that $\{\tilde{x}_{\lambda_n}^n\}$ is a soft Cauchy sequence. Suppose that $\{\tilde{x}_{\lambda_n}^n\}$ is not a Soft Cauchy sequence, which means that there is a constant $\epsilon_0 > 0$ such that for each positive integer k, there are positive integer $\lambda_{m(k)}$ and $\lambda_{n(k)}$ with $\lambda_{m(k)} > \lambda_{n(k)} > k$:

$$\tilde{d}\left(\tilde{x}_{\lambda_{m(k)}}^{m(k)}, \tilde{x}_{\lambda_{n(k)}}^{n(k)}\right) \geq \epsilon_0, \tilde{d}\left(\tilde{x}_{\lambda_{m(k)-1}}^{m(k)-1}, \tilde{x}_{\lambda_{n(k)}}^{n(k)}\right) < \epsilon_0$$

By triangle inequality

$$\epsilon_0 \leq \tilde{d}\left(\tilde{x}_{\lambda_{m(k)}}^{m(k)}, \tilde{x}_{\lambda_{n(k)}}^{n(k)}\right) \leq \tilde{d}\left(\tilde{x}_{\lambda_{m(k)}}^{m(k)}, \tilde{x}_{\lambda_{m(k)-1}}^{m(k)-1}\right) + \tilde{d}\left(\tilde{x}_{\lambda_{m(k)-1}}^{m(k)-1}, \tilde{x}_{\lambda_{n(k)}}^{n(k)}\right)$$
$$< \tilde{d}\left(\tilde{x}_{\lambda_{m(k)}}^{m(k)}, \tilde{x}_{\lambda_{m(k)-1}}^{m(k)-1}\right) + \epsilon_0$$

Letting $k \to \infty \lim_{k\to\infty} \tilde{d}\left(\tilde{x}_{\lambda_{m(k)}}^{m(k)}, \tilde{x}_{\lambda_{n(k)}}^{n(k)}\right) = \epsilon_0$**(**2.1.3)

Similarly, we have

$$\left.\begin{array}{c}\lim_{n\to\infty} \tilde{d}\left(\tilde{x}_{\lambda_{m(k)}}^{m(k)}, \tilde{x}_{\lambda_{n(k)+1}}^{n(k)+1}\right) = \epsilon_0, \lim_{n\to\infty} \tilde{d}\left(\tilde{x}_{\lambda_{n(k)}}^{n(k)}, \tilde{x}_{\lambda_{m(k)+1}}^{m(k)+1}\right) = \epsilon_0 \\ , \\ \lim_{n\to\infty} \tilde{d}\left(\tilde{x}_{\lambda_{m(k)+1}}^{m(k)+1}, \tilde{x}_{\lambda_{n(k)+1}}^{n(k)+1}\right) = \epsilon_0.\end{array}\right\}(2.1.4)$$

Putting $\tilde{x}_\lambda = \tilde{x}_{\lambda_{m(k)}}^{m(k)}$ and $\tilde{y}_\mu = \tilde{x}_{\lambda_{n(k)}}^{n(k)}$ in (2.1.1) we have

$$\widetilde{\mathcal{M}}\left(\tilde{x}_{\lambda_{m(k)}}^{m(k)}, \tilde{x}_{\lambda_{n(k)}}^{n(k)}\right) = \alpha \int_0^{\left\{\frac{\tilde{d}^3\left(\tilde{x}_{\lambda_{m(k)}}^{m(k)}, (f,\varphi)\left(\tilde{x}_{\lambda_{m(k)}}^{m(k)}\right)\right) + \tilde{d}^3\left(\tilde{x}_{\lambda_{n(k)}}^{n(k)}, (f,\varphi)\left(\tilde{x}_{\lambda_{n(k)}}^{n(k)}\right)\right)}{1+\tilde{d}^2\left(\tilde{x}_{\lambda_{m(k)}}^{m(k)}, (f,\varphi)\left(\tilde{x}_{\lambda_{m(k)}}^{m(k)}\right)\right) + \tilde{d}^2\left(\tilde{x}_{\lambda_{n(k)}}^{n(k)}, (f,\varphi)\left(\tilde{x}_{\lambda_{n(k)}}^{n(k)}\right)\right)}\right\}} \xi(t)dt$$

$$+ \gamma \int_0^{\left\{\tilde{d}\left(\tilde{x}_{\lambda_{m(k)}}^{m(k)}, \tilde{x}_{\lambda_{n(k)}}^{n(k)}\right)\right\}} \xi(t)dt$$

$$= \alpha \int_0^{\left\{\frac{\tilde{d}^3\left(\tilde{x}_{\lambda_{m(k)}}^{m(k)}, \tilde{x}_{\lambda_{m(k)+1}}^{m(k)+1}\right) + \tilde{d}^3\left(\tilde{x}_{\lambda_{n(k)}}^{n(k)}, \tilde{x}_{\lambda_{n(k)+1}}^{n(k)+1}\right)}{1+\tilde{d}^2\left(\tilde{x}_{\lambda_{m(k)}}^{m(k)}, \tilde{x}_{\lambda_{m(k)+1}}^{m(k)+1}\right) + \tilde{d}^2\left(\tilde{x}_{\lambda_{n(k)}}^{n(k)}, \tilde{x}_{\lambda_{n(k)+1}}^{n(k)+1}\right)}\right\}} \xi(t)dt$$

$$+ \gamma \int_0^{\left\{\tilde{d}\left(\tilde{x}_{\lambda_{m(k)}}^{m(k)}, \tilde{x}_{\lambda_{n(k)}}^{n(k)}\right)\right\}} \xi(t)dt$$

Letting $k \to \infty$ and using above equations we have

$\lim_{k\to\infty} \widetilde{\mathcal{M}}\left(\tilde{x}_{\lambda_{m(k)}}^{m(k)}, \tilde{x}_{\lambda_{n(k)}}^{n(k)}\right) = (\gamma)\epsilon_0 \ldots$(2.1.5)

$$\psi\left[\tilde{d}\left(\tilde{x}_{\lambda_{m(k)+1}}^{m(k)+1}, \tilde{x}_{\lambda_{n(k)+1}}^{n(k)+1}\right)\right] = \psi\left[\tilde{d}\left((f,\varphi)\left(\tilde{x}_{\lambda_{m(k)}}^{m(k)}\right), (f,\varphi)\left(\tilde{x}_{\lambda_{n(k)}}^{n(k)}\right)\right)\right]$$
$$\leq \psi\left[\widetilde{\mathcal{M}}\left(\tilde{x}_{\lambda_{m(k)}}^{m(k)}, \tilde{x}_{\lambda_{n(k)}}^{n(k)}\right)\right] - \varphi\left[\widetilde{\mathcal{M}}\left(\tilde{x}_{\lambda_{m(k)}}^{m(k)}, \tilde{x}_{\lambda_{n(k)}}^{n(k)}\right)\right]$$

Taking $k \to \infty$,and the continuity of ψ and φ ,we have

$$\psi[\epsilon_0] \leq \psi[(\gamma)\epsilon_0] - \varphi[(\gamma)\epsilon_0]$$
$$\leq \psi[\epsilon_0] - \varphi[(\gamma)\epsilon_0]$$

This leads to $\varphi[(\gamma)\epsilon_0] = 0$, and property of φ we get $\epsilon_0 = 0$.

This is a contradiction. Thus $\{\tilde{x}_{\lambda_n}^n\}$ is a soft Cauchy sequence in $\tilde{X}$, which is complete. Thus, there is $\tilde{x}_\lambda^* \in \tilde{X}$ such that $\tilde{x}_{\lambda_n}^n \to \tilde{x}_\lambda^*, n \to \infty$.

Taking $\tilde{x}_\lambda = \tilde{x}_{\lambda_n}^n$ and $\tilde{y}_\mu = \tilde{x}_\lambda^*$ in (4.1.2) we have

$$\widetilde{\mathcal{M}}(\tilde{x}^n_{\lambda_n},\tilde{x}^*_\lambda) = \alpha\int_0^{\left\{\frac{\tilde{d}^3\left(\tilde{x}^n_{\lambda_n},(f,\varphi)(\tilde{x}^n_{\lambda_n})\right)+\tilde{d}^3\left(\tilde{x}^*_\lambda,(f,\varphi)(\tilde{x}^*_\lambda)\right)}{1+\tilde{d}^2\left(\tilde{x}^n_{\lambda_n},(f,\varphi)(\tilde{x}^n_{\lambda_n})\right)+\tilde{d}^2\left(\tilde{x}^*_\lambda,(f,\varphi)(\tilde{x}^*_\lambda)\right)}\right\}}\xi(t)dt$$

$+\gamma\int_0^{\left\{\tilde{d}\left(\tilde{x}^n_{\lambda_n},\tilde{x}^*_\lambda\right)\right\}}\xi(t)dt\ldots$ (2.1.6)

Taking$n\to\infty$

$$\lim_{n\to\infty}\widetilde{\mathcal{M}}(\tilde{x}^n_{\lambda_n},\tilde{x}^*_\lambda)\leq(\alpha)\{\tilde{d}(\tilde{x}^*_\lambda,(f,\varphi)(\tilde{x}^*_\lambda))\}$$

$$\psi\left[\tilde{d}\left(\tilde{x}^{n+1}_{\lambda_{n+1}},(f,\varphi)(\tilde{x}^*_\lambda)\right)\right]=\psi\left[\tilde{d}\left((f,\varphi)(\tilde{x}^n_{\lambda_n}),(f,\varphi)(\tilde{x}^*_\lambda)\right)\right]$$

$$\leq\psi[\widetilde{\mathcal{M}}(\tilde{x}^n_{\lambda_n},\tilde{x}^*_\lambda)]-\varphi[\widetilde{\mathcal{M}}(\tilde{x}^n_{\lambda_n},\tilde{x}^*_\lambda)]$$

$\psi[\tilde{d}(\tilde{x}^*_\lambda,(f,\varphi)(\tilde{x}^*_\lambda))]\leq\psi[(\alpha)\int_0^{\{\tilde{d}(\tilde{x}^*_\lambda,(f,\varphi)(\tilde{x}^*_\lambda))\}}\xi(t)dt]$-

$$-\varphi[(\alpha)]\int_0^{\{\tilde{d}(\tilde{x}^*_\lambda,(f,\varphi)(\tilde{x}^*_\lambda))\}}\xi(t)dt$$

$\psi[\tilde{d}(\tilde{x}^*_\lambda,(f,\varphi)(\tilde{x}^*_\lambda))]\leq\psi\int_0^{\tilde{d}(\tilde{x}^*_\lambda,(f,\varphi)(\tilde{x}^*_\lambda))}\xi(t)dt-\varphi[\alpha\int_0^{\{\tilde{d}(\tilde{x}^*_\lambda,(f,\varphi)(\tilde{x}^*_\lambda))\}}\xi(t)dt]$

Which implies $\varphi[(\alpha)\{\tilde{d}(\tilde{x}^*_\lambda,(f,\varphi)(\tilde{x}^*_\lambda))\}]=0$,

So $\tilde{d}(\tilde{x}^*_\lambda,(f,\varphi)(\tilde{x}^*_\lambda))=0$,that is$(f,\varphi)(\tilde{x}^*_\lambda)=\tilde{x}^*_\lambda$.

Uniqueness: Let$\tilde{y}^*_\mu$is another fixed point of(f,φ)in$\tilde{X}$such that$\tilde{x}^*_\lambda\neq\tilde{y}^*_\mu$, then

Putting $\tilde{x}_\lambda=\tilde{x}^*_\lambda$ and $\tilde{y}_\mu=\tilde{y}^*_\lambda$

$\widetilde{\mathcal{M}}(\tilde{x}^*_\lambda,\tilde{y}^*_\lambda) = \alpha\int_0^{\left\{\frac{\tilde{d}^3(\tilde{x}^*_\lambda,(f,\varphi)(\tilde{x}^*_\lambda))+\tilde{d}^3(\tilde{y}^*_\lambda,(f,\varphi)(\tilde{y}^*_\lambda))}{1+\tilde{d}^2(\tilde{x}^*_\lambda,(f,\varphi)(\tilde{x}^*_\lambda))+\tilde{d}^2(\tilde{y}^*_\lambda,(f,\varphi)(\tilde{y}^*_\lambda))}\right\}}\xi(t)dt+\gamma\int_0^{\{\tilde{d}(\tilde{x}^*_\lambda,\tilde{y}^*_\lambda)\}}\xi(t)dt$

$$\psi[\tilde{d}(\tilde{x}^*_\lambda,\tilde{y}^*_\lambda)]=\psi[\tilde{d}((f,\varphi)(\tilde{x}^*_\lambda),(f,\varphi)(\tilde{y}^*_\lambda))]$$

$$\leq\psi[\widetilde{\mathcal{M}}(\tilde{x}^*_\lambda,\tilde{y}^*_\lambda)]-\varphi[\widetilde{\mathcal{M}}(\tilde{x}^*_\lambda,\tilde{y}^*_\lambda)]$$

$$\leq\psi[(\gamma)\tilde{d}(\tilde{x}^*_\lambda,\tilde{y}^*_\lambda)]-\varphi[(\gamma)\tilde{d}(\tilde{x}^*_\lambda,\tilde{y}^*_\lambda)]$$

$$\psi[\tilde{d}(\tilde{x}^*_\lambda,\tilde{y}^*_\lambda)]\leq\psi[\tilde{d}(\tilde{x}^*_\lambda,\tilde{y}^*_\lambda)]-\varphi[(\gamma)\tilde{d}(\tilde{x}^*_\lambda,\tilde{y}^*_\lambda)]$$

So$\varphi[(\gamma)\tilde{d}(\tilde{x}^*_\lambda,\tilde{y}^*_\lambda)]=0$,thus$\tilde{d}(\tilde{x}^*_\lambda,\tilde{y}^*_\lambda)=0$,that is$\tilde{x}^*_\lambda=\tilde{y}^*_\lambda$.

Hence fixed point of$(f,\varphi)$is unique.

Corollary4.2: Let$(\tilde{X},\tilde{d},E)$be a soft complete metric space. Suppose the soft mapping$(f,\varphi):(\tilde{X},\tilde{d},E)\to(\tilde{X},\tilde{d},E)$satisfies the following condition:

$\psi\left[\tilde{d}\left((f,\varphi)(\tilde{x}_\lambda),(f,\varphi)(\tilde{y}_\mu)\right)\right]\leq\psi[\widetilde{\mathcal{M}}(\tilde{x}_\lambda,\tilde{y}_\mu)]-\varphi[\widetilde{\mathcal{M}}(\tilde{x}_\lambda,\tilde{y}_\mu)]$(2.2.1)

For each $\tilde{x}_\lambda,\tilde{y}_\mu\in\tilde{X},\tilde{x}_\lambda\neq\tilde{y}_\mu$,where ψ,φ are altering distance functions, and

$\widetilde{\mathcal{M}}(\tilde{x}_\lambda,\tilde{y}_\mu)=\alpha\int_0^{\left\{\frac{\tilde{d}^3(\tilde{x}^*_\lambda,(f,\varphi)(\tilde{x}^*_\lambda))+\tilde{d}^3(\tilde{y}^*_\lambda,(f,\varphi)(\tilde{y}^*_\lambda))}{1+\tilde{d}^2(\tilde{x}^*_\lambda,(f,\varphi)(\tilde{x}^*_\lambda))+\tilde{d}^2(\tilde{y}^*_\lambda,(f,\varphi)(\tilde{y}^*_\lambda))}\right\}}\xi(t)dt+\gamma\int_0^{\{\tilde{d}(\tilde{x}^*_\lambda,\tilde{y}^*_\lambda)\}}\xi(t)dt$

$$+\delta\int_0^{\left\{\tilde{d}(\tilde{x}_\lambda,(f,\varphi)(\tilde{x}_\lambda))+\tilde{d}(\tilde{y}_\mu,(f,\varphi)(\tilde{y}_\mu))\right\}}\xi(t)dt$$

$$+\eta\int_0^{\left\{\tilde{d}(\tilde{x}_\lambda,(f,\varphi)(\tilde{y}_\mu))+\tilde{d}(\tilde{y}_\mu,(f,\varphi)(\tilde{x}_\lambda))\right\}}\xi(t)dt$$

Where $\alpha,\gamma,\delta,\eta>0and2\alpha+\gamma+2\delta+2\eta<1$is a soft constant. Then (f,φ)has a unique soft fixed point in$\widetilde{X}$.

Proof: It can be proved easily as previous theorem

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