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A Common Random Fixed-Point Theorems in Intuitionistic Fuzzy Metric Spaces

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Abstract - A common random fixed-point theorem is established in this paper. The obtained results are the modified form of well-known results of fixed-point theory. Results are established for intuitionistic fuzzy metric space for rational contraction through integral type mappings.

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Key words: Intuitionistic fuzzy metric space (IFMS), weakly Compatible Mappings (WCM), Common Random fixed point (CRFP).

I. INTRODUCTION

The idea of fuzzy set was given by Zadeh [15].After the notion of intuitionistic fuzzy sets introduced and studied byAtanassov [14].Coker [6] presented the idea of intuitionistic fuzzy topological spaces.Further, Jungck [13] proved some common fixed point theorem in intuitionistic fuzzy metric space.Later on, Turkoglu et al. [7] formulated the notions of weakly commuting and R weakly commuting mappings in intuitionistic fuzzy metric spaces and obtained the intuitionistic fuzzy version of Pant's theorem [16]. In fewrecent years, the study of random fixed point have attracted much attention. In particular, random iteration schemes leading to random fixed point of random operator. The present paper deals with some fixed point theorems for random operator in IFMS. We find unique random fixed point of random operator by considering a sequence of measurable functions satisfying particular conditions.

II. PRELIMINARIES

Throughout this paper (Ω, Σ)denotes a measurable space consisting of a set Ω and sigma algebra Σ of subset of Ω . X stands for a Banach space, and C is non empty subset of X.

Definition 2A: A function R : $\Omega \times C \to C$is said to be measurable if

$R^{-1}(B \cap C) \in \Sigma$for every Borel subset B of X.

Definition 2B: A function R : $\Omega \times C \to C$ is said to be random operator, if

R(., x) : $\Omega \to C$ is measurable for every $x \in C$.

Definition 2C: A random operator R : $\Omega \times C \to C$is said to be continuous if for fixed $\xi \in \Omega, R(\xi, .) : C \to C$is continuous.

Definition 2D:. A measurable function $g : \Omega \to C$is said to be random fixed point of the random operator R : $\Omega \times C \to C$, if $R\left(\xi, g(\xi)\right) = g(\xi)$, □ξ ∈ Ω□ □ □ Or R$g(\xi)) = g(\xi)$.

Definition 2.1[modified in 1]: Let (X, d) be a complete metric space, c∈ (0, 1) and f: X→X be a mapping such that for each x, y ∈ X, ξ is a measerable selector and (Ω, Σ)denotes a measurable space consisting of a set Ω and sigma algebra Σ of subset of Ω . A: $\tilde{X} \to \tilde{X}$

$$\int_0^{d(f\xi x, f\xi y)} \varphi(t)dt \le c \int_0^{d(\xi x, \xi y)} \varphi(t)dt$$

where φ: [0,+∞) →[0,+∞) is a Lebesgue integrable mapping which is summable on each compact subset of [0,+∞) , non-negative, and such that for each ε> o, $\int_0^{\varepsilon} \varphi(t)dt$,then f has a unique fixed point $a \in X$ such that for each $\xi x \in X$, $\lim_{n\to\infty} f^n \xi x = \xi a$.

B.E.Rhoades [2],extended the result of Branciari [1] by replacing the above contractive condition by the following

$$\int_0^{d(f\xi x, f\xi y)} \varphi(t)dt$$
$$\le c \int_0^{max\left\{d(\xi x,\xi y), d(\xi x, f\xi x), d(\xi y, f\xi y), \frac{d(\xi x, f\xi y)+d(\xi y, f\xi x)}{2}\right\}} \varphi(t)dt.$$

Definition 2.2[3].A binary operation *: [0, 1] ×[0, 1] →[0, 1] is a continuous t-norm if,

(i) * is associative and commutative,

(ii) * is continuous,

(iii) a * 1 = a for all a ∈[0, 1],

(iv) a * b ≤ c * d whenever a ≤ c and b ≤ d for all a, b, c, d ∈[0, 1],

Two typical examples of continuous t-norm are a * b = ab and a * b = min (a, b).

Definition 2.3[3]. A binary operation ✧: [0, 1] ×[0, 1] →[0, 1] is a continuous t-norm if

(i) ✧ is associative and commutative,

(ii) ✧ is continuous,

(iii) a✧1 = a for all a ∈[0, 1],

(iv) a✧b ≤ c ✧d whenever a ≤ c and b ≤ d for all a, b, c, d ∈[0, 1],

Two typical examples of continuous t-co norm are a✧b = ab and a ✧b = min (a, b).

Definition 2.4[4]. A 5-tuple (X, M, N, ∗ ,✧) is said to be an intuitionistic fuzzy metric space with random operator, if X is an arbitrary set, ∗ is a continuous t-norm, ✧ is a continuous t-co-norm and M, N are fuzzy sets on $X^2 \times [0,\infty)$, ξ is a measerable selector and (Ω,Σ) denotes a measurable space consisting of a set Ω and sigma algebra Σ of subset of Ω . A: $\tilde{X} \to \tilde{X}$ satisfying the following conditions:

(i) $M(\xi x, \xi y, t) + N(\xi x, \xi y, t) \leq 1 \, for\ all\ \xi x, \xi y \in X\ and\ t > 0$;

(ii) $M(\xi x, \xi y, 0) = 0\ for\ all\ \xi x, \xi y \in X$;

(iii) $M(\xi x, \xi y, t) = 1\ for\ all\ \xi x, \xi y \in X\ and\ t > 0\ if\ and\ only\ if\ \xi x = \xi y$;

(iv) $M(\xi x, \xi y, t) = M(\xi y, \xi x, t)\ for\ all\ \xi x, \xi y \in X\ and\ t > 0$;

(v) $M(\xi x, \xi y, t) * M(\xi y, \xi z, \xi s) \leq M(\xi x, \xi z, t + s\ for\ all\ \xi x, \xi y \in X\ and\ s, t > 0$;

(vi) $for\ all\ \xi x, \xi y \in X, M(\xi x, \xi y, .): [0, \infty) \to [0,1]\ is\ left\ continuous$;

(vii) $\lim_{t\to\infty} M(\xi x, \xi y, t) = 1\ for\ all\ \xi x, \xi y \in X\ and\ t > 0$;

(viii) $N(\xi x, \xi y, 0) = 1\ for\ all\ \xi x, \xi y \in X$;

(ix) $M(\xi x, \xi y, t) = 1\ for\ all\ \xi x, \xi y \in X\ and\ t > 0\ if\ and\ only\ if\ \xi x = \xi y$;

(x) $N(\xi x, \xi y, t) = N(\xi y, \xi x, t)\ for\ all\ \xi x, \xi y \in X\ and\ t > 0$;

(xi) $N(\xi x, \xi y, t) \diamond N(\xi y, \xi z, s) \geq N(\xi x, \xi z, t + s\ for\ all\ \xi x, \xi y \in X\ and\ s, t > 0$;

(xii) $for\ all\ \xi x, \xi y \in X, N(x, y, .): [0, \infty) \to [0,1]\ is\ right\ continuous$;

(xiii) $\lim_{t\to\infty} N(\xi x, \xi y, t) = 0\ for\ all\ \xi x, \xi y \in X\ and\ t > 0$.

Then (M, N) is called an intuitionistic fuzzy metric space on X with random operator. The functions M(x, y, t) and N(x, y, t) denote the degree of nearness and the degree of non-nearness between x and y w.r.t. t respectively.

Remark 2.1[modified in 4]. Every fuzzy metric space $(X, M, *)$ is an intuitionistic fuzzy metric space of the form $(X, M, 1-M, *, \diamond)$ such that t-norm ∗ and t-co norm ✧defined by $a * a \geq a, a \in [0,1]$&$(1-a) \diamond (1-a) \leq (1-a) for\ all\ x, y \in X$,In IFM space $(X, M, N, *, \diamond, M\xi x, \xi y, *$ is non-decreasing and $N\xi x, \xi y, \diamond$ is non-increasing with random operator.

Remark 2.2[Modified in 17]. Let (X, d) be a metric space. Define t-norm a * b = min (a, b) andt-co norm a ✧b = max (a, b), $for\ all\ \xi x, \xi y \in X\ \&\ t > 0$.

$M_d(\xi x, \xi y, t) = \frac{t}{t + d(\xi x, \xi y)}$, $N_d(\xi x, \xi y, t) = \frac{d(\xi x, \xi y)}{t + d(\xi x, \xi y)}$.Then $(X, M, N, *, \diamond)$ is an intuitionistic fuzzy metric space induced by the metric. It is obvious that $N(\xi x, \xi y, t) = 1 - M(\xi x, \xi y, t)$

[Modified in 4]

Definition 2.5. Let (X, M, N, ∗ ,✧) be an intuitionistic fuzzy metric space ξ is a measerable selector and (Ω,Σ) denotes a measurable space consisting of a set Ω and sigma algebra Σ of subset of Ω . A: $\tilde{X} \to \tilde{X}$,Then

(i) a sequence $\{\xi x_n\}$ in X is called Cauchy-sequence if, for all $t > 0\ \&\ P > 0$, $\lim_{n\to\infty} M(\xi x_{n+p}, \xi x_n, t) = 1$and $\lim_{n\to\infty} N(\xi x_{n+p}, \xi x_n, t) = 0$,

(ii) a sequence $\{\xi x_n\}$ in X is said to be convergent to a point $\xi x \in X$ if , for all $t > 0$,

$\lim_{n\to\infty} M(\xi x_n, \xi x, t) = 1$ and $\lim_{n\to\infty} N(\xi x_n, \xi x, t) = 0$.

Definition 2.6. Let (X, M, N, ∗ ,✧) be an intuitionistic fuzzy metric space and$\{\xi y_n\}$ be a sequence in X if there exists a number $k \in (0,1)$, ξ is a measerable selector and (Ω,Σ) denotes a measurable space consisting of a set Ω and sigma algebra Σ of subset of Ω . A: $\tilde{X} \to \tilde{X}$

such that:

1. $M(\xi y_{n+2}, \xi y_{n+1}, kt) \geq M(\xi y_{n+1}, \xi y_n, t)$,

2. $N(\xi y_{n+2}, \xi y_{n+1}, kt) \leq N(\xi y_{n+1}, \xi y_n, t)$

for all t > 0 and n = 1 ,2 ,3……then $\{y_n\}$ is a Cauchy sequence in X.

Definition 2.7. A pair of self-mappings (f, g) of an intuitionistic fuzzy metric space

(X, M, N, ∗ ,✧) is said to be compatible if $\lim_{n\to\infty} M(fg\xi x_n, gf\xi x_n, t) = 1$&$\lim_{n\to\infty} N(fg\xi x_n, gf\xi x_n, t) = 0$ for every t > 0, whenever $\{\xi x_n\}$ is a sequence in X such that $\lim_{n\to\infty} f\xi x_n = \lim_{n\to\infty} g\xi x_n = \xi z$, for some.$\xi z \in X$.

Definition 2.8. A pair of self-mappings (f, g) of an intuitionistic fuzzy metric space

(X, M, N, $*$,◊) is said to be non-compatible if $\lim_{n\to\infty} M(fg\xi x_n, gf\xi x_n, t) \neq 1$&$\lim_{n\to\infty} N(fg\xi x_n, gf\xi x_n, t) \neq 0$ for every t > 0, whenever $\{\xi x_n\}$ is a sequence in X such that $\lim_{n\to\infty} f\xi x_n = \lim_{n\to\infty} g\xi x_n = \xi z$, for some.$\xi z \in X$.

Definition 2.9. An intuitionistic fuzzy metric space (X, M, N, $*$,◊) is said to be complete if and only if every Cauchy sequence in X is convergent.

Lemma 2.1. Let (X, M, N, $*$,◊) be an intuitionistic fuzzy metric space and $for\ all\ \xi x, \xi y \in X, t > 0$ and if for a number $k \in (0,1)$ such that $M(\xi x, \xi y, kt) \geq M(\xi y, \xi x, t)$& $N(\xi x, \xi y, kt) \leq N(\xi y, \xi x, t)$then $\xi x = \xi y$.

Definition 2.9[modification in 13]. A pair of self-mappings (f, g) of a metric space is said to be weakly compatible for random operator ξ if they commute at their coincidence points i.e. $f\xi u = gu\ for\ some\ \xi u \in X$,then $fg\xi u = gf\xi u$.

3. Main results

Theorem 3.1. Let A, B, S and T be self-maps of intuitionistic fuzzy metric spaces

(X, M, N, $*$,◊) with continuous t-norm * and continuous t-co norm ◊defined by $t * t \geq t$ and $(1-t) \diamond (1-t) \leq (1-t)$ for all $t \in [0,1]$, ξ is a measerable selector and (Ω, Σ)denotes a measurable space consisting of a set Ω and sigma algebra Σ of subset of Ω . A: $\tilde{X} \to \tilde{X}$satisfying the following condition:

(3.1.1) A(X) ⊆S(X) and B(X) ⊆T(X),

(3.1.2) If one of the A, B, S and T is a complete subspace of X then {A, T}&{B, S} have a coincidence point,

(3.1.3) The pairs (A, T) and (B, S) are weakly compatible,

(3.1.4)

$$\int_0^{M(A\xi x, B\xi y, t)} \varphi(t)dt \geq \int_0^{\emptyset\left\{min\begin{pmatrix} M(T\xi x, S\xi y, t) * M(T\xi x, A\xi x, t) * M(A\xi x, S\xi y, t) * \\ M(S\xi y, T\xi x, t) * M(B\xi x, T\xi y, t) * M(B\xi x, S\xi x, t)\end{pmatrix}\right\}} \varphi(t)dt$$

and

$$\int_0^{N(A\xi x, B\xi y, t)} \varphi(t)dt \leq \int_0^{\varphi\left\{max\begin{pmatrix} N(T\xi x, \boldsymbol{S\xi y}, t) \diamond N(T\xi x, A\xi x, t) \diamond N(A\xi x, S\xi y, t) \diamond \\ N(S\xi y, T\xi x, t) \diamond N(B\xi x, T\xi y, t) \diamond N(B\xi x, S\xi x, t)\end{pmatrix}\right\}} \varphi(t)dt$$

$\forall \xi x, \xi y \in X$&$> 0$, where $\emptyset, \varphi: [0,1] \to [0,1]$ is a continuous function such that $\emptyset(t) > t$&$\varphi(t) < t$ for each 0 < t < 1 and $\emptyset(1) = 1$ and $\varphi(0) = 0$ with $M(\xi x, \xi y, t) > 0$.

Then A, B, S and T have a unique common random fixed point in X.

Proof: Since A(X) ⊆S(X), therefore for any $x_0 \in X$, there exists a point $\xi x_1 \in X$ such that $A\xi x_0 = S\xi x_1$ and for the point ξx_1 , we can choose a point $x\xi_2 \in X$ such that $B\xi x_1 = T\xi x_2$ as B(X) ⊆S(X). Inductively, we getSequence $\{\xi y_n\}$ in x as follows $\xi y_{2n+1} = B\xi x_{2n+1} = T\xi x_{2n+2}$ and $\xi y_{2n} = A\xi x_{2n} = S\xi x_{2n+1}$ for n = 0,1,2….. Putting $x = \xi x_{2n}$, $y = x\xi_{2n+1}$ in (3.1.4) we have,

$$\int_0^{M(A\xi x_{2n}, B\xi x_{2n+1}, t)} \varphi(t)dt \geq \int_0^{\emptyset\left\{min\begin{pmatrix} M(T\xi x_{2n}, S\xi x_{2n+1}, t) * M(T\xi x_{2n}, A\xi x_{2n}, t) * \\ M(A\xi x_{2n}, S\xi x_{2n+1}, t) * M(S\xi x_{2n+1}, T\xi x_{2n}, t) * \\ M(B\xi x_{2n}, T\xi x_{2n+1}, t) * M(B\xi x_{2n}, S\xi x_{2n}, t)\end{pmatrix}\right\}} \varphi(t)\mathrm{dt}$$

$$\int_0^{M(\xi y_{2n}, \xi y_{2n+1}, t)} \varphi(t)dt \geq \int_0^{\emptyset\left\{min\begin{pmatrix} M(\xi y_{2n-1}, \xi y_{2n}, t) * M(\xi y_{2n-1}, \xi y_{2n}, t) * \\ M(\xi y_{2n}, \xi y_{2n}, t) * M(\xi y_{2n}, \xi y_{2n-1}, t) * \\ M(\xi y_{2n-1}, \xi y_{2n}, t) * M(\xi y_{2n-1}, \xi y_{2n}, t)\end{pmatrix}\right\}} \varphi(t)dt$$

$$\int_0^{M(\xi y_{2n}, \xi y_{2n+1}, t)} \varphi(t)dt \geq \int_0^{\emptyset\left\{min\begin{pmatrix} M(\xi y_{2n-1}, \xi y_{2n}, t) * M(\xi y_{2n-1}, \xi y_{2n}, t) * \\ 1 * M(\xi y_{2n}, \xi y_{2n-1}, t) * \\ M(\xi y_{2n-1}, \xi y_{2n}, t) * M(\xi y_{2n-1}, \xi y_{2n}, t)\end{pmatrix}\right\}} \varphi(t)dt$$

$$\int_0^{M(\xi y_{2n},\xi y_{2n+1},t)} \varphi(t)dt \geq \int_0^{\emptyset\{M(\xi y_{2n-1},\xi y_{2n},t)\}} \varphi(t)dt > \int_0^{M(\xi y_{2n-1},\xi y_{2n},t)} \varphi(t)dt$$

as $\emptyset(t) > t$ for each $0 < t < 1$ and

$$\int_0^{N(A\xi x_{2n},B\xi x_{2n+1},t)} \varphi(t)dt \leq \int_0^{\varphi\left\{max\begin{pmatrix} N(T\xi x_{2n},S\xi x_{2n+1},t)\diamond N(T\xi x_{2n},A\xi x_{2n},t)\diamond \\ N(A\xi x_{2n},S\xi x_{2n+1},t)\diamond N(S\xi x_{2n+1},T\xi x_{2n},t)\diamond \\ N(B\xi x_{2n},T\xi x_{2n+1},t)\diamond N(B\xi x_{2n},S\xi x_{2n},t)\end{pmatrix}\right\}} \varphi(t)dtdt$$

$$\int_0^{N(\xi y_{2n},\xi y_{2n+1},t)} \varphi(t)dt \leq \int_0^{\varphi\left\{max\begin{pmatrix} N(\xi y_{2n-1},\xi y_{2n},t)\diamond N(\xi y_{2n-1},\xi y_{2n},t)\diamond \\ N(\xi y_{2n},\xi y_{2n},t)\diamond N(\xi y_{2n},\xi y_{2n-1},t)\diamond \\ N(\xi y_{2n-1},\xi y_{2n},t)\diamond N(\xi y_{2n-1},y_{2n},t)\end{pmatrix}\right\}} \varphi(t)dt$$

$$\int_0^{N(\xi y_{2n},\xi y_{2n+1},t)} \varphi(t)dt \leq \int_0^{\varphi\left\{max\begin{pmatrix} N(\xi y_{2n-1},\xi y_{2n},t)\diamond N(\xi y_{2n-1},\xi y_{2n},t)\diamond \\ 1\diamond N(\xi y_{2n},\xi y_{2n-1},t)\diamond \\ N(\xi y_{2n-1},\xi y_{2n},t)\diamond N(\xi y_{2n-1},\xi y_{2n},t)\end{pmatrix}\right\}} \varphi(t)dt$$

$$\int_0^{N(\xi y_{2n},\xi y_{2n+1},t)} \varphi(t)dt \leq \int_0^{\varphi\{N(\xi y_{2n-1},\xi y_{2n},t)\}} \xi(t)dt < \int_0^{N(\xi y_{2n-1},\xi y_{2n},t)} \varphi(t)dt$$

as $\varphi(t) < t$ for each $0 < t < 1$.

Thus $\{M(\xi y_{2n},\xi y_{2n+1},t), n \geq 0\}$ is an increasing sequence of positive real numbers in [0,1] which tends to a limit $l \leq 1$, also $\{N(\xi y_{2n+1},\xi y_{2n+2},t), n \geq 0\}$ is an decreasing sequence of positive real numbers [0 ,1] which tends to a limit $k = 0$.Therefore for every $n \in I^+ M(\xi y_n,\xi y_{n+1},t) > M(\xi y_{n-1},\xi y_n,t)$& $\lim_{n\to\infty} M(\xi y_n,\xi y_{n+1},t) = 1$,$N(\xi y_n,\xi y_{n+1},t)N(\xi y_{n-1},\xi y_n,t)$&$\lim_{n\to\infty} N(\xi y_n,\xi y_{n+1},t) = 0$.

Now any positive integer p, we obtain

$$\lim_{n\to\infty} M(\xi y_n,\xi y_{n+p},t) = 1 \; \& \; \lim_{n\to\infty} N(\xi y_n,\xi y_{n+p},t) = 0$$

Which shows that $\{\xi y_n\}$ is a Cauchy sequence in X. Let $w \in S^{-1}\xi u$then $S\xi w = \xi u$. we shall use the fact that subquence $\{y_{2n+1}\}$ also converges to u.Now by putting $x = x_{2n}$, $y = w$ in (3.1.4) and taking $n \to \infty$

$$\int_0^{M(A\xi x_{2n},B\xi w,t)} \varphi(t)dt \geq \int_0^{\emptyset\left\{min\begin{pmatrix} M(T\xi x_{2n},S\xi w,t)*M(T\xi x_{2n},A\xi x_{2n},t)* \\ M(A\xi x_{2n},S\xi w,t)*M(S\xi w,T\xi x_{2n},t)* \\ M(B\xi x_{2n},T\xi w,t)*M(B\xi x_{2n},S\xi x_{2n},t)\end{pmatrix}\right\}} \varphi(t)dt$$

$$\int_0^{M(\xi u,B\xi w,t)} \varphi(t)dt \geq \int_0^{\emptyset\left\{min\begin{pmatrix} M(\xi u,\xi u,t)*M(\xi u,\xi u,t)*M(\xi u,\xi u,t)* \\ M(\xi u,\xi u,t)*M(\xi u,\xi u,t)*M(\xi u,\xi u,t)\end{pmatrix}\right\}} \varphi(t)dt$$

$$\int_0^{M(\xi u,B\xi w,t)} \varphi(t)dt \geq \int_0^{\emptyset\{M(\xi u,\xi u,t)\}} \xi(t)dt \geq \int_0^{\emptyset(1)} \varphi(t)dt$$

i.e $M(\xi u, B\xi w, t) \geq 1 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots. (*)$

also

$$\int_0^{N(A\xi x_{2n},B\xi w,t)} \varphi(t)dt \leq \int_0^{\varphi\left\{max\begin{pmatrix} N(T\xi x_{2n},S\xi w,t)\diamond N(T\xi x_{2n},A\xi x_{2n},t)\diamond \\ N(A\xi x_{2n},S\xi w,t)\diamond N(S\xi w,T\xi x_{2n},t)\diamond \\ N(B\xi x_{2n},T\xi w,t)\diamond N(B\xi x_{2n},S\xi x_{2n},t)\end{pmatrix}\right\}} \varphi(t)dt$$

$$\int_0^{N(\xi u,B\xi w,t)} \varphi(t)dt \leq \int_0^{\varphi\left\{max\begin{pmatrix} N(\xi u,\xi u,t)\diamond N(\xi u,\xi u,t)\diamond N(\xi u,\xi u,t)\diamond \\ N(\xi u,\xi u,t)\diamond N(\xi u,\xi u,t)\diamond N(\xi u,\xi u,t)\end{pmatrix}\right\}} \varphi(t)dt$$

$$\int_0^{N(\xi u,B\xi w,t)} \varphi(t)dt \le \int_0^{\varphi\{N(\xi u,\xi u,t)\}} \varphi(t)dt \le \int_0^{\varphi(0)} \varphi(t)dt$$

i.e $N(\xi u, B\xi w, t) \le 0 \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots .. (**)$

From (*) and (**), Let $\xi u = B\xi w$. Since $S\xi w = \xi u$ we have $S\xi w = B\xi w = \xi u$ i.e. ξw is the coincidence point of B and S. As B(X) ⊆T(X), $= B\xi w \to \xi u \,\epsilon\, T(X)$. Let $\xi v \in T^{-1}\xi u$then $T\xi v = \xi u$. Now by putting $\xi x = \xi v$, $\xi y = \xi x_{2n+1}$ in (3.1.4)

$$\int_0^{M(A\xi v,B\xi x_{2n+1},t)} \varphi(t)dt \ge \int_0^{\emptyset\left\{min\begin{pmatrix} M(T\xi v,S\xi x_{2n+1},t)*M(T\xi v,A\xi v,t)* \\ M(A\xi v,S\xi x_{2n+1},t)*M(S\xi x_{2n+1},T\xi v,t)* \\ M(B\xi v,T\xi x_{2n+1},t)*M(B\xi v,S\xi v,t)\end{pmatrix}\right\}} \varphi(t)dt$$

taking $n \to \infty$

$$\int_0^{M(A\xi v,\xi u,t)} \varphi(t)dt \ge \int_0^{\emptyset\left\{min\begin{pmatrix} M(\xi u,\xi u,t)*M(\xi u,A\xi v,t)*M(A\xi v,\xi u,t)* \\ M(\xi u,\xi u,t)*M(\xi u,\xi u,t)*M(\xi u,\xi u,t)\end{pmatrix}\right\}} \varphi(t)dtt$$

$$\int_0^{M(A\xi v,\xi u,t)} \varphi(t)dt \ge \int_0^{\emptyset\{min\,(1*M(\xi u,A\xi v,t)*M(A\xi v,\xi u,t)*1*1*1)\}} \varphi(t)dt$$

$$i.e \int_0^{M(A\xi v,\xi u,t)} \varphi(t)dt \;\ge \int_0^{\emptyset\{M(\xi u,A\xi v,t)\}} \varphi(t)dt > \int_0^{M(\xi u,A\xi v,t)} \varphi(t)dt$$

and

$$\int_0^{N(A\xi v,B\xi x_{2n+1},t)} \varphi(t)dt \le \int_0^{\varphi\left\{max\begin{pmatrix} N(T\xi v,S\xi x_{2n+1},t)\diamond N(T\xi v,A\xi v,t)\diamond \\ N(A\xi v,S\xi x_{2n+1},t)\diamond N(S\xi x_{2n+1},T\xi v,t)\diamond \\ N(B\xi v,T\xi x_{2n+1},t)\diamond N(B\xi v,S\xi v,t)\end{pmatrix}\right\}} \varphi(t)dt$$

taking $n \to \infty$

$$\int_0^{N(A\xi v,\xi u,t)} \varphi(t)dt \le \int_0^{\varphi\left\{max\begin{pmatrix} N(\xi u,\xi u,t)\diamond N(\xi u,A\xi v,t)\diamond N(A\xi v,\xi u,t)\diamond \\ N(\xi u,\xi u,t)\diamond N(\xi u,\xi u,t)\diamond N(\xi u,\xi u,t)\end{pmatrix}\right\}} \varphi(t)dt$$

$$\int_0^{N(A\xi v,\xi u,t)} \varphi(t)dt \le \int_0^{\varphi\{max\,(1\diamond N(\xi u,A\xi v,t)\diamond N(A\xi v,\xi u,t)\diamond 1\diamond 1\diamond 1)\}} \varphi(t)dt$$

$$i.e \int_0^{N(A\xi v,\xi u,t)} \varphi(t)dt \le \int_0^{\emptyset\{N(\xi u,A\xi v,t)\}} \varphi(t)dt < \int_0^{N(\xi u,A\xi v,t)} \varphi(t)dt$$

Therefore, we get $A\xi v = \xi u$. we have $T\xi v = A\xi v = \xi u$. Thus ξv is a coincidence point of A & T.

Since the pairs $\{A,T\}$ *and* $\{B,S\}$ are weakly compatible i.e. $B(S\xi w) = S(B\xi w) \to B\xi u = S\xi u$ and $A(T\xi v) = T(A\xi v) \to A\xi u = T\xi u$.Now by putting $\xi x = \xi u$, $\xi y = \xi x_{2n+1}$ in

$$\int_0^{M(A\xi u,B\xi x_{2n+1},t)} \varphi(t)dt \;\ge \int_0^{\emptyset\left\{min\begin{pmatrix} M(T\xi u,S\xi x_{2n+1},t)*M(T\xi u,A\xi u,t)* \\ M(A\xi u,S\xi x_{2n+1},t)*M(S\xi x_{2n+1},T\xi u,t)* \\ M(B\xi u,T\xi x_{2n+1},t)*M(B\xi u,S\xi u,t)\end{pmatrix}\right\}} \varphi(t)dt$$

taking $n \to \infty$ (3.1.4)

$$\int_0^{M(A\xi u,\xi u,t)} \varphi(t)dt \geq \int_0^{\emptyset\left\{min\begin{pmatrix} M(A\xi u,\xi u,t)*M(A\xi u,A\xi u,t)*M(A\xi u,\xi u,t)* \\ M(\xi u,A\xi u,t)*M(\xi u,\xi u,t)*M(\xi u,\xi u,t)\end{pmatrix}\right\}} \varphi(t)dt$$

$$\int_0^{M(A\xi u,\xi u,t)} \varphi(t)dt \geq \int_0^{\emptyset\{min\,(M(A\xi u,\xi u,t)*1*M(A\xi u,\xi u,t)*M(\xi u,A\xi u,t)*1*1)\}} \varphi(t)dt$$

$$i.e \int_0^{M(A\xi u,\xi u,t)} \varphi(t)dt \geq \int_0^{\emptyset\{M(A\xi u,\xi u,t)\}} \xi(t)dt > \int_0^{M(A\xi u,\xi u,t)} \varphi(t)dt$$

and

$$\int_0^{N(A\xi u,B\xi x_{2n+1},t)} \varphi(t)dt \leq \int_0^{\varphi\left\{max\begin{pmatrix} N(T\xi u,S\xi x_{2n+1},t)\diamond N(T\xi u,A\xi u,t)\diamond \\ N(A\xi u,S\xi x_{2n+1},t)\diamond N(S\xi x_{2n+1},T\xi u,t)\diamond \\ N(B\xi u,T\xi x_{2n+1},t)\diamond N(B\xi u,S\xi u,t)\end{pmatrix}\right\}} \varphi(t)dt$$

taking $n \to \infty$

$$\int_0^{N(A\xi u,\xi u,t)} \varphi(t)dt \leq \int_0^{\varphi\left\{max\begin{pmatrix} N(A\xi u,\xi u,t)\diamond N(A\xi u,A\xi u,t)\diamond N(A\xi u,\xi u,t)\diamond \\ N(\xi u,A\xi u,t)\diamond N(\xi u,\xi u,t)\diamond N(\xi u,\xi u,t)\end{pmatrix}\right\}} \varphi(t)dt$$

$$\int_0^{N(A\xi u,\xi u,t)} \varphi(t)dt \leq \int_0^{\varphi\{max\,(N(A\xi u,\xi u,t)\diamond 1\diamond N(A\xi u,\xi u,t)\diamond N(\xi u,A\xi u,t)\diamond 1\diamond 1)\}} \varphi(t)dt$$

$$i.e \int_0^{N(A\xi u,\xi u,t)} \varphi(t)dt \leq \int_0^{\varphi\{N(A\xi u,\xi u,t)\}} \xi(t)dt < \int_0^{N(A\xi u,\xi u,t)} \varphi(t)dt$$

Therefore, we get $A\xi u = \xi u$. So we have $A\xi u = T\xi u = \xi u$. similarly by putting $\xi x = \xi x_{2n}$, $\xi y = \xi u$ in (3.1.4) as $n \to \infty \xi u = B\xi u = S\xi u$. Thus $A\xi u = B\xi u = S\xi u = T\xi u = \xi u$ i.e. ξu is a common fixed point of A, B, S and T.

Uniqueness: Let $\xi w(\xi w \neq \xi u)$ be another common fixed point of A, B, S and T. then by putting $\xi x = \xi u , \xi y = \xi w$

in (3.1.4)

$$\int_0^{M(A\xi u,B\xi w,t)} \varphi(t)dt \geq \int_0^{\emptyset\left\{min\begin{pmatrix} M(T\xi u,S\xi w,t)*M(T\xi u,A\xi u,t)*M(A\xi u,S\xi w,t)* \\ M(S\xi w,T\xi u,t)*M(B\xi u,T\xi w,t)*M(B\xi u,S\xi u,t)\end{pmatrix}\right\}} \varphi(t)dt$$

$$\int_0^{M(\xi u,\xi w,t)} \varphi(t)dt \geq \int_0^{\emptyset\left\{min\begin{pmatrix} M(\xi u,\xi w,t)*M(\xi u,\xi u,t)*M(\xi u,\xi w,t)* \\ M(\xi w,\xi u,t)*M(\xi u,\xi w,t)*M(\xi u,\xi u,t)\end{pmatrix}\right\}} \varphi(t)dt$$

$$\int_0^{M(\xi u,\xi w,t)} \varphi(t)dt \geq \int_0^{\emptyset\left\{min\begin{pmatrix} M(\xi u,\xi w,t)*1*M(\xi u,\xi w,t)*M(\xi w,\xi u,t)* \\ M(u,w,t)*1\end{pmatrix}\right\}} \varphi(t)dt$$

$$i.e \int_0^{M(\xi u,\xi w,t)} \varphi(t)dt \geq \int_0^{\emptyset\{M(\xi u,\xi w,t)\}} \xi(t)dt > \int_0^{M(\xi u,\xi w,t)} \varphi(t)dt$$

and

$$\int_0^{N(A\xi u,B\xi w,t)} \varphi(t)dt \leq \int_0^{\varphi\left\{max\begin{pmatrix} N(T\xi u,S\xi w,t)\diamond N(T\xi u,A\xi u,t)\diamond N(A\xi u,S\xi w,t)\diamond \\ N(S\xi w,T\xi u,t)\diamond N(B\xi u,T\xi w,t)\diamond N(B\xi u,S\xi u,t)\end{pmatrix}\right\}} \varphi(t)dt$$

$$\int_0^{N(\xi u,\xi w,t)} \varphi(t)dt \leq \int_0^{\varphi\left\{max\begin{pmatrix} N(\xi u,\xi w,t)\diamond N(\xi u,\xi u,t)\diamond N(\xi u,\xi w,t)\diamond \\ N(\xi w,\xi u,t)\diamond N(\xi u,\xi w,t)\diamond N(\xi u,\xi u,t)\end{pmatrix}\right\}} \varphi(t)dt$$

$$\int_0^{N(\xi u,\xi w,t)} \varphi(t)dt \leq \int_0^{\varphi\left\{max\begin{pmatrix} N(\xi u,\xi w,t)\diamond 1\diamond N(\xi u,\xi w,t)\diamond N(\xi w,\xi u,t)\diamond \\ N(\xi u,\xi w,t)\diamond 1\end{pmatrix}\right\}} \varphi(t)dt$$

$$i.e \int_0^{N(\xi u,\xi w,t)} \xi\varphi(t)dt \leq \int_0^{\varphi\{N(\xi u,\xi w,t)\}} \xi(t)dt < \int_0^{N(\xi u,\xi w,t)} \varphi(t)dt$$

Hence $\xi u = \xi w$ for all $\xi x\ , y\xi \in X\ and\ t > 0$.

Therefore ξu is the unique common random fixed point of A, B, S and T.

This completes the proof.

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