

High Rate and Full Diversity 2x2 STC with Low Complexity Reception

Modepalli Prathap¹, S. Ramkumar², T. Venkatrao³

^{1,2,3}Assist. Prof. L.M.A.T, Vijayawada

Abstract - 2×2 MIMO profiles included in Mobile Wi-MAX specifications are Alamouti's STC (space-time code) for transmit diversity and spatial multiplexing (SM). Alamouti's STC has full diversity and the SM has full rate, but neither of these has both of desired features as required. Golden code provides full rate and full diversity. It has a high decoding complexity. The issue decoding complexity was included in the STC design criteria, and variant STCs were proposed. In this paper, a high-rate full-diversity 2×2 STC design leading to lower complexity of the optimum detector compared to the Golden code with only a slight performance loss. We provided the general optimized form of this STC and show that this scheme achieves the diversity multiplexing for different QAM signal constellations.

Keywords - STC, MIMO, Wi-MAX.

I. INTRODUCTION

Multiple-Input Multiple-Output (MIMO) concepts have been under development for many years for wireless systems. Digital communication using MIMO is emerging as one of the most promising research areas in wireless communications. The MIMO approach can also be used for spatial diversity rather than multiplexing to benefit the Bit Error Rate (BER) performance of the wireless communication channel. A full-rate full-diversity 2×2 STC design leading to substantially lower complexity of the optimum detector compared to the Golden code with only a slight performance loss.

At the system level, careful design of MIMO signal processing and coding algorithms can help increase coverage. Today, MIMO wireless is widely recognized as one of three or four key technologies in coming high-speed high-spectrum efficiency wireless networks (4G, and to some extent 3G).

Wireless system designers are facing a number of challenges. Major of them are limited availability of the radio frequency spectrum and space, time varying wireless channel. Further, there is an increasing demand for higher data rates, better QOS(Quality of Service) and higher network capacity. In

recent years, MIMO systems have come as promising technology in these limits. Effective technique to provide reliable communication over a wireless channel is diversity which attempts to provide the receiver with independently faded copies of the transmitted signal with expectations to at least one of these replicas will be received correctly.

One of the main advantages of MIMO systems is the substantial increase in the channel capacity, higher data throughputs. Another advantage of MIMO systems is low symbol error rates. These advantages are made without any expansion in the bandwidth or increase in the transmit power. As information is transmitted through different paths, a MIMO system utilizes transmitter and receiver diversity techniques, hence maintaining reliable communications.

II. FRFD 2X2 STC CODE DESIGN CRITERIA

A finite set of complex matrices is a STBC. A $n \times n$ linear STBC is obtained starting from an $n \times n$ matrix consisting of arbitrary linear combinations of m complex variables and conjugates of those symbols, and it leads to variables take values from complex constellations. The rate of such a code is k/n complex symbols per channel use. We consider Rayleigh quasi-static flat fading MIMO channel with full channel state information (CSI) at the receiver but not at the transmitter. For 2×2 MIMO transmission, we have

$$Y = HS + N \quad (2.1)$$

Code rate: If there are k independent information symbols in the codeword which are transmitted over T channel uses, then, for an $n_t \times n_r$ MIMO system, the code rate is defined as k/T symbols per channel. If $k = n_{\min} T$, where $n_{\min} = \min(n_t, n_r)$, then the STBC is said to have full rate.

Considering ML decoding, the decoding metric that is to be minimized over all possible values of code words S is given by

$$M(S) = \|Y - HS\|^2 \quad (2.2)$$

for, number of symbols = 4, Time slots =2, (4/2) = 2. Symbols for each channel used are 2 and with 2 transmit, 2 receive antennas $n_{min} = \min(n_t, n_r) = 2$, shows full rate.

Decoding complexity: The ML decoding complexity (number of metrics) is given by the minimum number of symbols that need to be jointly decoded in minimizing the decoding metric. This can never be greater than k , in which case, the decoding complexity is said to be of the order of Mk . If the decoding complexity is lesser than Mk , the code is said to simplified complex decoding. If, constellation size is 16 then metrics computed are $(16^4) = 256$.

III. PROPOSED LOW COMPLEXITY FRFD STC

I present my approach to full-rate 2x2 STC design makes to maximize both the diversity gain while leading to an optimum detector of reduced complexity depends on constellation. More specifically, the proposed STC is a high-rate, full-diversity 2x2 space-time code whose optimum receiver has a complexity that is only proportional to M^2 , where M is the size of the signal constellation. The number of Euclidean distance computations(calculations) in the optimum detector is reduced to $16^2 = 256$ for a 16-QAM signal constellation and to $64^2 = 4,096$ for a 64-QAM signal group. By comparing these information to those connected to the Golden code (or Matrix C), it becomes obvious that this code makes the implementation of full-rate, full-diversity 2x2 STCs with optimum receiver practical. We currently present a general picture of the proposed code. A group of 4 data symbols (s_1, s_2, s_3, s_4) in the proposed code design is transmitted as follows:

$$X_{new} = \begin{bmatrix} as_1 + bs_3 & -cs_2^* - ds_4^* \\ as_2 + bs_4 & cs_1^* + ds_3^* \end{bmatrix} \quad (3.5)$$

where $a, b, c,$ and d are parameters (complex valued design parameters) and the star symbol indicates complex conjugate.

In this matrix representation, the first column represents the symbol combinations transmitted during a first symbol interval t_1 and the second column represents the symbol combinations transmitted during a second symbol interval t_2 . The initial row of the matrix give the sign combinations transmitted from the first Tx antenna, and second row of the matrix gives the symbol combinations transmitted from the second Tx antenna. In other words, $a_{s_1} + b_{s_3}$ is transmitted from Tx antenna 1 during the first symbol interval t_1 , $a_{s_2} + b_{s_4}$ is transmitted from Tx antenna 2 during the first symbol interval t_1 , $-c_{s_2}^* - d_{s_4}^*$ is transmitted from Tx antenna 1 during the second symbol interval t_2 , and $c_{s_1}^* + d_{s_3}^*$ is

transmitted from Tx antenna 2 during the second symbol interval t_2 .

On the first receive antenna, the two signals received at the first and second symbol intervals are:

$$r_1 = h_{11} (a_{s_1} + b_{s_3}) + h_{12} (a_{s_2} + b_{s_4}) + n_1, \quad (3.6)$$

$$r_2 = h_{11} (-c_{s_2}^* - d_{s_4}^*) + h_{12} (c_{s_1}^* + d_{s_3}^*) + n_2, \quad (3.7)$$

Similarly, we have on the second Rx antenna:

$$r_3 = h_{21} (a_{s_1} + b_{s_3}) + h_{22} (a_{s_2} + b_{s_4}) + n_3, \quad (3.8)$$

$$r_4 = h_{21} (-c_{s_2}^* - d_{s_4}^*) + h_{22} (c_{s_1}^* + d_{s_3}^*) + n_4. \quad (3.9)$$

where n_i , for $i = 1, \dots, 4$, are the additive noise terms.

The maximum likelihood (ML) detector makes an exhaustive search over all possible values of the transmitted symbols and decides in favor of the quadruplet (S_1, S_2, S_3, S_4) which minimizes the Euclidean distance:

$$D(s_1, s_2, s_3, s_4) = \left\{ \left| r_1 - h_{11}(a_{s_1} + b_{s_3}) - h_{12}(a_{s_2} + b_{s_4}) \right|^2 + \left| r_2 - h_{11}(-c_{s_2}^* - d_{s_4}^*) - h_{12}(c_{s_1}^* + d_{s_3}^*) \right|^2 + \left| r_3 - h_{21}(a_{s_1} + b_{s_3}) - h_{22}(a_{s_2} + b_{s_4}) \right|^2 + \left| r_4 - h_{21}(-c_{s_2}^* - d_{s_4}^*) - h_{22}(c_{s_1}^* + d_{s_3}^*) \right|^2 \right\} \quad (3.10)$$

An exhaustive search clearly involves the computation of M^4 metrics and $M^4 - 1$ comparisons, which is excessive for the 16-QAM and 64-QAM signal constellations. But the proposed STC design lends itself to a low-complexity implementation of the ML detector as we now show.

From the received signal samples (r_1, r_2, r_3, r_4), let us compute w_1, w_2, w_3, w_4 signals:

From those signals, we next compute the signal Y_1 given by:

$$y_1 = (h_{11}^* w_1 + h_{21}^* w_3) / a + (h_{12} w_2^* + h_{22} w_4^*) / c^* \\ = (|h_{11}|^2 + |h_{12}|^2 + |h_{21}|^2 + |h_{22}|^2) s_1 + \eta_1 \quad (3.11)$$

$$\text{with } \eta_1 = (h_{11}^* n_1 + h_{21}^* n_3) / a + (h_{12} n_2^* + h_{22} n_4^*) / c^*$$

It can be seen that the signal y_1 has no terms involving symbol s_2 , and the coefficient of the term in s_1 clearly indicates that estimation of s_1 benefits from 4th-order detector, we get the ML estimate of symbol s_1 conditional on (s_3, s_4). Note that the elimination of the terms involving s_2 is possible if and only if the respective coefficients of the

symbols s_1 and s_2 in each column of the code matrix are identical.

This method, which is shown in Fig. 2, reduces the ML receiver complexity from M^4 to M^2 .

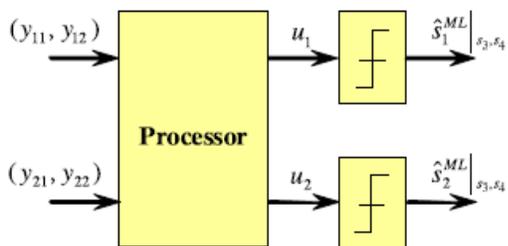


Fig. 1 Operation of the received signals to determine the ML estimate of symbols s_1 and s_2 conditional on a particular combination of symbols s_3 and s_4 .

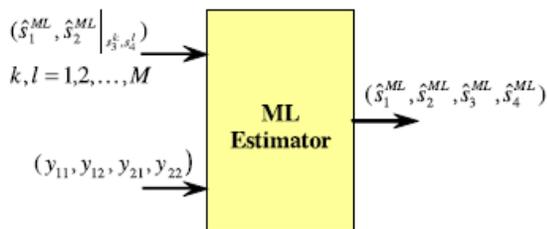


Fig. 2 Second stage of the estimator.

Note that the special structure of figure1 allows the ML detector also to work the other way round: Instead of deriving the ML estimate of (s_1, s_2) conditional on (s_3, s_4) and then computing the metric $D(s_1, s_2, s_3, s_4)$ for $(s_1^k, s_2^l, s_3^k, s_4^l)$, we can first estimate (s_3, s_4) conditional on (s_1^k, s_2^l) , then compute the metric $D(s_1, s_2, s_3, s_4)$ for $(s_1^k, s_2^l, s_3^{ML}, s_4^{ML})$, for $k, l = 1, 2, \dots, M$, and select the quadruplet (s_1, s_2, s_3, s_4) minimizing the metric.

It is instructive to point out here that the described detector is optimum only when the magnitudes of a and c (alternatively the magnitudes of b and d for the reverse detection order) are equal. This can be easily seen by looking at the SNR at the receiver input and then at the threshold detection input. Certainly, these two signal to noise ratio values are the same if and only if $|a| = |c|$ for forward detection and $|b| = |d|$ for reverse detection.

The a, b, c, d parameters in the code matrix are design parameters to be optimized in order to obtain full-diversity STC with large coding gain. This task takes long time for higher constellation sizes. The transmit power constraints can further decrease the number of parameters to be optimized.

In terms of the transmitted power, conditions can expressed as

$$|a|^2 + |b|^2 = |c|^2 + |d|^2$$

$$|a|^2 + |c|^2 = |b|^2 + |d|^2$$

The first condition ensures an equal transmit power at all symbol instance, as the next condition ensures that equal whole power is transmitted for each symbol. These equalities together with the constraint $|a| = |c|$ for optimal detection lead immediately to the fact that all the design parameters should have the same scale, i.e., $|a| = |c| = |b| = |d| = 0.707$.

Without any loss of generality, we take $a=c = 0.707$ (this allows to decrease the number of unknown parameters without affecting the coding gain) and make an exhaustive search to optimize the parameters b and d .

A. Rate- $3/4 \times 2 \times 2$ STC

The STC given in (5.5) can be modified for a further reduction in the optimum detector complexity. More specifically, by setting $s_4 = s_3$ and scaling the energy of this symbol, we obtain the following 2×2 code with rate $3/4$:

$$X_{new}^{3/4} = \begin{bmatrix} as_1 + bs_3/\sqrt{2} & -cs_2^* - ds_3^*/\sqrt{2} \\ as_2 + bs_3/\sqrt{2} & cs_1^* + ds_3^*/\sqrt{2} \end{bmatrix}, (3.21)$$

where the notation $X_{new}^{3/4}$ is used to distinguish the proposed code $X_{new}(5.5)$ from its reduced-rate version.

In order to detect the transmitted symbols, the occupied ML detector makes an thorough search over all possible values of the transmitted symbols and decides in favor of the triplet (s_1, s_2, s_3) which minimizes the Euclidean distance that we denote by $D(s_1, s_2, s_3)$. Specifically, this exhaustive search involves the computation of M^3 metrics and $M^3 - 1$ comparisons, which is also excessive for the 16-QAM and 64-QAM signal combinations. Currently, falling the symbol s_4 lends itself to a lower-complexity implementation of the ML detector at the price of transmission rate reduction.

More precisely, following the same procedure as that presented for the full-rate case, it can be seen that the signals $u_k, k = 1, 2$, will have only terms involving the respective symbol s_k and the estimation of symbols $s_k, k = 1, 2$, will benefit from full fourth-order spatial diversity. By sending the signals u_1 and u_2 to a threshold detector, we acquire the ML estimation of symbol s_1 and s_2 conditional only on the mark s_3 . Make a note that, as a natural consequence of similarity to the full-rate case, the elimination of the terms

Involving s_2 can be possible iff the coefficients a and c have the same scale. In this method, for a known value of symbol s_3 , we get the ML estimate of (s_1, s_2) , which we denote $(s_1^{ML}, s_2^{ML} | s_3)$. Now, instead of computing the metric $D(s_1, s_2, s_3)$ for all (s_1, s_2, s_3) values, we only need to compute it for $((s_1^{ML}, s_2^{ML} | s_3), s_3)$. In other words, the optimum receiver computes the metric $D(s_1, s_2, s_3)$ for $((s_1^{ML}, s_2^{ML} | s_3^l), s_3^l), l = 1, \dots, M$. This procedure evidently reduces the ML receiver complexity from M^3 to M . Optimization of the parameters in the reduced-rate case can be performed similarly to the full-rate case. The parameters a and c can be set to $1/\sqrt{2}$ without any loss of generality.

IV. RESULTS

In this section, we present some comparisons between the new STCs and the existing STCs. Simulations were carried out for different QPSK, 16-QAM and 64-QAM signal constellations, and the results are calculated for an uncorrelated Rayleigh fading channel with $E[|h_{kl}|^2] = 1$ for all k, l . Only 2 receive antennas were used in all cases.

A. Complexity, Rate and Diversity comparison

Complexity is determined by number of metrics computed for symbol estimation. Table 1 shows Rate $3/4$ code is least complex than existing STCs because it requires only four metrics with QPSK constellation, sixteen(16) metrics with 16-QAM constellation and sixty four(64) metrics with 64-QAM constellation, we can't treat full rate(only 3 symbols are transmitted in two time slots in stead of four). In general Rate $3/4$ code complexity is M (constellation size).

	QPSK	16 QAM	64 QAM	Rate	Diversity
Alamouti STC	16	256	4096	1/2(half)	Full (4th order)
SM	16	256	4096	1(full)	Half (2nd order)
Golden Code	256	65536	16777216	1(full)	Full (4th order)
New FRFD STC(low)	16	256	4096	1(full)	Full (4th order)
Rate 3/4	4	16	64	3/4	full

Table 1: Complexity(number of metrics), corresponding Rate and Diversity comparison for different STCs

Complexity is determined by number of metrics computed for symbol estimation. As shown in table 1 Rate $3/4$ code is www.ijspr.com

least complex Existing STC's it requires only four(4) metrics with QPSK constellation, sixteen(16) metrics with 16-QAM constellation and 64 metrics with 64-QAM constellation, it is not full rate(as we transmitting three(3) symbols are transmitted in two time slots). In general Rate $3/4$ code complexity is M (indicates constellation size). So in practical implementation of view Rate $3/4$ code is implementable with less hardware(less chip area),even it is possible for higher constellations 512-QAM, 1024-QAM. Practical implementation becomes expensive for Golden Code even it is full rate and full diversity STC at higher constellation sizes.

B. Performance Comparison in the Full-Rate Case

Performance comparisons between the low complexity full-rate full diversity 2×2 STC and the Alamouti STC. Figure 4 shows the BER performance as a function of E_b/N_0 , where E_b denotes the average signal energy per symbol, and provides comparisons between X_{new} , new STC, and X_g is a Golden code. It can be seen that X_{new} achieves the same diversity gain and gives essentially the same results as X_g at substantially lower complexity. Indeed, their conclusion is that the performance of low complexity full-rate full diversity 2×2 STC is marginally very close to that of X_g .

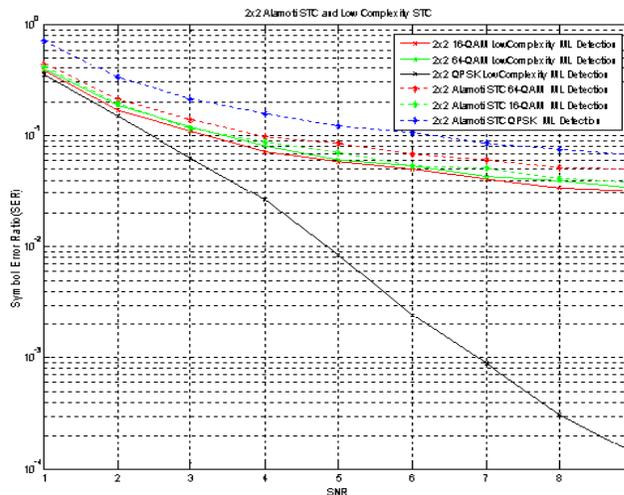


Fig. 3 Performance variation between low complexity HRFD STC and Alamouti STC

The complexity reduction can be observed from table 1, low complexity High-rate full diversity 2×2 STC results in a considerable reduction in the number of computations. These results indicate that X_{new} enables to reduce the hardware complexity without any significant performance degradation.

C. Performance Comparison in the Rate-3/4 Case

We now provide a performance comparison between $X_{3/4}^{new}$, the proposed rate-3/4 STC, and the two MIMO schemes in current mobile WiMAX system specifications (Alamouti's STC and SM). With the optimized values, the proposed STC maximizes the diversity gain as indicated in figure 3 and, therefore, it achieves the same BER slope of curve as Alamouti's STC with constant coding gain independent of the constellation size.

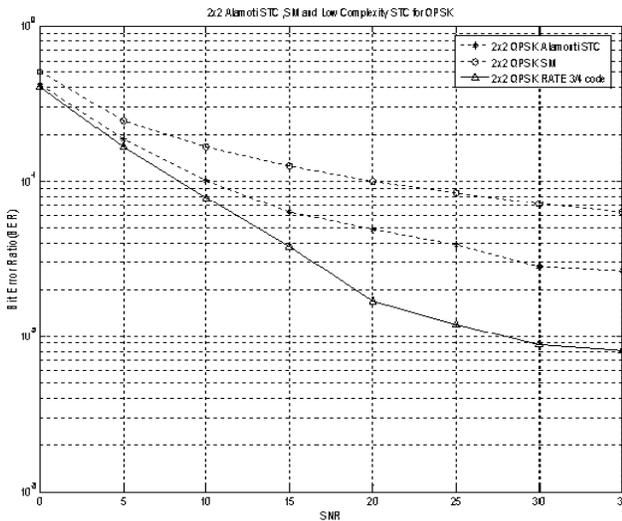


Fig. 4 SNR for various QPSK constellations of Alamouti, SM and Rate3/4 STCs

The results shown in Figure 4 indicate that, the Alamouti scheme has the best BER performance and also the lowest bit rate on a given channel. The SM scheme doubles the bit rate, but we find strong SNR loss, SNR loss increases at lower BER(Bit Error Rate) values. As Shown from these results, the proposed rate-3/4 scheme is an interesting alternative to those two MIMO schemes.

V. CONCLUSION AND FUTURE SCOPE

In this paper, I presented a new low complexity high-rate full-diversity 2×2 STC leading to a low-complexity optimum decoder. We have compared its performance with existing codes and the results shows that the proposed scheme achieves good performance of the known code while reducing the decoder complexity by magnitude in QPSK, 16-QAM, and 64-QAM based MIMO systems. The rate-3/4 and full-diversity 2×2 STC whose optimum decoder complexity increases linearly with the number of constellations. We have compared its performance to the two MIMO schemes, As the results are showing that it stands as an interesting alternative providing further good performance and spectral efficiency.

In future the low complexity STC design will implement for higher order 4×4 , 6×6 and various antenna configurations as described above. Thus, the STC design creates new perspectives for next evolutions of Wi-MAX systems and for other wireless systems.

REFERENCES

- [1] IEEE 802.16-2005: IEEE Standard for Local and Metropolitan Area Networks.
- [2] S.M.Alamouti, "A simple transmit diversity technique for wireless communications," IEEE J. Select. Areas Commun., vol. 16, pp. 1451-1458, Oct. 1998.
- [3] J.-C. Belfiore, G. Rekaya, and E. Viterbo, "The Golden code: a 2×2 full-rate space-time code with non-vanishing determinants," IEEE Trans. Inform. Theory, vol. 51, pp. 1432-1436, Apr. 2005.
- [4] D. Tse and P. Viswanath, Fundamentals of Wireless Communications. Cambridge University Press, 2005.
- [5] S. Sezginer and H. Sari, "A full-rate full-diversity 2×2 space-time code for mobile WiMAX systems," in Proc. ICSPC'07, Dubai, UAE, Nov. 2007.
- [6] J.-C. Guey, M. P. Fitz, M. R. Bell, and W.-Y. Kuo, "Signal design for transmitter diversity wireless communications systems over Rayleigh fading channels," in Proc. VTC'96 Spring, 1996, pp. 136-140.
- [7] V. Tarokh, N. Seshadri, and A. R. Calderbank, "Space-time codes for high data rate wireless communications: performance criterion and code construction," IEEE Trans. Inform. Theory, vol. 44, pp. 744-765, Mar. 1998.
- [8] J. Paredes, A. B. Gershman, and M. G. Alkhanari, "A 2×2 spacetime code with non-vanishing determinants and fast maximum likelihood decoding," in Proc. ICASSP'07, Hawaii, USA, Apr. 2007.