New Type Mixed Duality in Multi Objective Fractional Programming Under Generalized ρ-Convexity Function

Gayatri Devi, Rashmita Mohanty
Prof. CSE, ABIT College, CDA-1, Cuttack

Abstract : Two New types of mixed duality are introduced in this paper. Weak and strong duality theorems are established under generalized ρ -convexity. Also established necessary and sufficient optimality condition.

Keywords : Non differentiable fractional programming, symmetric duality, generalized convexity,ρ-function.

I. INTRODUCTION

A fractional programming problem arises in many types of optimization problem such as portfolio selection, production, information theory and numerous decision making problems in management science.

Multi objecive fractional programming duality has been of much interest in the recent part. Schaible [1] and Bectar et.al. [2] derived Fritz John and Karush-Kuhn Tucker necessary, and sufficient optimality condition for a class of non-differentiable convex multi objective fractional programming problems and established duality theorems. Bectar et.al. [3] and Xu [4] gave a mixed type duality for fractional programming, established some duality results.

Several authors, such as the ones of [5, 6, 7, 8, 9], studied multi objective non-differentiable multi objective fractional problem in which numerators contains support function.

Motivated by the earlier authors in this paper we introduced new type of mixed dual of a non differentiable multi objective fractional programming ;problem using generalized ▯-convex assumptions. Also we established the necessary and sufficient optimality condition. Section 3.5 deals with conclusion and scope for future work.

II. NOTATIONS AND PRELIMINARIES

Let R^n be the n-dimensional Euclidean space and R_+^n be its non –negative orthant. The following conventions for inequality will be used in this paper. For any x = (x_1, x_2, …., x_n), y = (y_1, y_2, ….,y_n), we denote

(i) $x > y \Leftrightarrow x_i > y_i$ for all $i = 1, z, ...n$

(ii) $x \geq y \Leftrightarrow x_i \geq y_i$ and $x \neq y$

Let x be a non empty open subset of R^n.

Consider the following non differentiable multiobjective fractional programming problem :

For N = {1, 2, …n}, let $J_1 \subset N$ and $J_2 = N / J_1$.

Let $|J_1|, |J_2|$ denote the number of elements in the set J_1 and J_2.

If $J_1 = \phi$, then $J_2 = N$ that is $|J_1| = 0$ and $|J_2| = n$

Hence $R^{|J_1|}$ is zero dimensional Euclidean space and $R^{|J_2|}$ is n-dimensional Euclidean space.

It is clear that any $x \in R^n$ can be written as

$$x = (x^1, x^2), x^1 \in R^{|J_1|}, x^2 \in R^{|J_2|}$$

Let $\frac{f_1}{g_1} : R^{|J_1|} \to R^\ell$ and $\frac{f_2}{g_2} : R^{|J_2|} \to R^\ell$ be twice differentiable functions and $e = (1, 1, ...1)^T \in R^\ell$

(MFP0) minimize F(x) = f(x) / g(x)

$$(\mathbf{MFP1}) = F_1(x^1) + F_2(x^2)$$

$$= \left[\left(f_1(x^1) - \nu_1 g_1(x^1)\right) + \left(f_2(x^2) - \nu_2 g_2(x^2)\right)\right] + ... + \lim_{x \to \infty}$$

$$\left[\left(f_{1\ell}(x^1) - \nu_{1\ell} g_{1\ell}(x^1)\right) + \left(f_{2\ell}(x^2) - \nu_{2\ell} g_{2\ell}(x^2)\right)\right] \text{ for } i = l,....,\ell.$$

(MFP_2) minimize $\lambda\left(F_1(x^1) + F_2(x^2)\right)$, λ is ℓ-dimensional strictly positive vector

$$= \lambda F(x)$$

Subject to $h(x) \le 0$

$h(x) = h_i j(x^1) + h_{2j}(x^2),\ j = (1, 2, ...m)$ are differentiable functions $h_{1j} : R^{|J_1|} \to R,\ h_{2j} : R^{|J_2|} \to R,$

$x^1 \in R^{|J_1|},\ x^2 \in R^{|J_2|}$

We assume that $f_i(x) \ge 0$ and $g_i(x) > 0$ on R^n for I = 1, 2, …. k.

Let $x_0 = \{x \in X \subseteq R^n : h_j(x) \le 0,\ j = 1, 2, ...m\}$ for feasible of MFP1 and denote I = {1, 2, ….., k}, M = M = {1, 2, ….. m}, $J_1 = \{j \in M;\ h_j(x) = 0\}$ and $J_2 = \{j \in M;\ h_j(x) < 0\}$. It is obvious that $J_1 \cup J_2 = M$

In the following definition let $f_i : X \to R,\ \eta : X \times X \to R^n,\ \rho \in R_+$

Definition 2.1 : f is said to be □-invex at $\bar{x} \in X$ with respect to □□□□□ if

$$f_i(x) - f_i(\bar{x}) \ge \eta(x, \hat{x})^T \nabla f_i(x) + \rho\|x - \bar{x}\|^2,\ \forall x \in X$$

Definition 2.2 : f is said to be □-psudo invex at $\bar{x} \in X$ with respect to □□□□□if

$$\eta(x, \bar{x})^T \nabla f_i(x) + \rho\|x - \bar{x}\|^2 \ge 0$$

$$\Rightarrow f_i(x) - f_i(\bar{x}) \ge 0\ \forall\ x \in X$$

Definition 2.3 : f_i is said to be □-quasiinvex at $\bar{x} \in X$ with respect to □□□□□if

$$f_i(x) - f_i(\bar{x}) \le 0$$

$$\Rightarrow \eta(x, \bar{x})^T \nabla f_i(x) + \rho\|x - \bar{x}\|^2 \le 0, \forall x \in X.$$

Definition 2.4 : A feasible point $\bar{x}$ is said to be efficient for (MFPO) if there exist no other feasible point *x* in (MFPO) such that

$F_i(x) \le F_i(\bar{x}),\ i = 1, 2, ...k$ and $F_r(x) < F_r(\bar{x})$ for some $r \in (1, 2, ...k)$

Definition 2.5 : A feasible point $\bar{x}$ is said to be properly efficient for (MFPO), if it is efficient and there exist M > 0 such that for each I = (1, 2, … k) and for all feasible point *x* in (MFPO) satisfying $F_i(x) < F_i(\bar{x})$, we have

$F_i(\bar{x}) - F_i(x) \le M(F_r(x) - F_r(\bar{x}))$ for some r such that $F_r(x) > F_r(\bar{x})$.

III. LEMMA

Lemma 3.1 : If x^0 is an optimal solution of MFP1 then x^0 is properly efficient for MFPO.

Lemma 3.2 : If x^0 is an efficient solution for MFPO iff it is an efficient solution of MFP1 with $F(x^0) = 0$.

Lemma 3.3 : (Necessary optimal condition)

If $\bar{x} \in X$ is an optimal solution of (MFPO) such that

$$\nabla\lambda F(\bar{x}) + y^t \nabla h(\bar{x}) = \sum_{i=1}^{k} \lambda_i \left[\nabla f_i(\bar{x}) - \nu_i \nabla g_i(\bar{x})\right] + \sum_{i=1}^{m} y_i \nabla h_i(\bar{x}) = 0 \quad [3.3.1]$$

$$F_i(\bar{x}) = f_i(\bar{x}) - \nu_i g_i(\bar{x}) = 0 \quad [3.3.2]$$

$$y^T h(\bar{x}) = 0 \quad [3.3.3]$$

$$y \ge 0 \quad [3.3.4]$$

$$\nu_i \ge 0,\ i = 1, 2, ...\ k \quad [3.3.5]$$

Lemma 3.4 : (Sufficient optimality condition)

Let $\bar{x} \in X^0$ be a feasible solution of (MFPO) and there exist $\lambda_i \in R_+$, $\nu_i \in R_+$ and $\bar{y} \in R^m$ satisfying the condition in Lemma 3.1 at $\bar{x}$.

Furthermore suppose that any one of the condition (a) or (b) holds

(a) $\rho(x) = \sum_{i=1}^{k} \lambda_i \left[f_i(x) - \nu_i g_i(x) \right] + \sum_{j=1}^{m} y_j h_j(x)$ is □-pseudo invex with respect to □□□□□at $\bar{x} \in X^0$

(b) $Q(x) = \sum_{i=1}^{k} \lambda_i \left[f_i(x) - \nu_i g_i(x) \right]$ is □-pseudo invex and $H(x) = \sum_{j=1}^{m} y_j h_j(x)$ is □-quasi invex with respect to ρ, η at $\bar{x} \in X^0$. Then $\bar{x}$ is an efficient solution of (MFPO).

Proof :

Suppose hypothesis (a) holds.

Since the conditions of Lemma 3.3.3 are satisfied, from 3.3.1, we have $\nabla p(\bar{x}) = 0$. So for $\eta(x, \bar{x}) \in R^n$, we can write $\eta(x, \bar{x})^T \nabla p(\bar{x}) = 0$. For $\rho \in R_T$, we have $\eta(x, \bar{x})^T \nabla p(\bar{x}) + \rho \left\| x - \bar{x} \right\|^2 \geq 0$

Since p(x) is □-pseudo invex with respect to η, ρ at $\bar{x} \in X^0$, we have $\left(p(x) - p(\bar{x}) \right) \geq 0$ and

$$\sum_{i=1}^{k} \lambda_i \left[f_i(x) - \nu_i g_i(x) \right] + \sum_{j=1}^{m} y_j h_j(x) \geq \sum_{i=1}^{k} \lambda_i \left[f_i(\bar{x}) - \nu_i g_i(\bar{x}) \right] + \sum_{j=1}^{m} y_j h_j(\bar{x}) \tag{3.3.6}$$

Suppose $\bar{x}$ is not efficient solution of MFP1, then there exist $x \in X^0$ such that

$f_i(x) - \nu_i g_i(x) \leq f_i(\bar{x}) - \nu_i g_i(\bar{x}), i = 1, 2, ...k$ and $f_i(x) - \nu_i g_i(x) < f_i(\bar{x}) - \nu_i g_i(\bar{x})$ for some i $\in$ (1, 2, …k). The above relation together with the relation $\bar{\lambda}_i > 0$ implies that

$$\sum_{i=1}^{k} \lambda_i \left[f_i(x) - \nu_i g_i(x) \right] < \sum_{i=1}^{k} \lambda_i \left[f_i(\bar{x}) - \nu_i g_i(\bar{x}) \right] \tag{3.3.7}$$

From the relation (3.2.1), (3.2.1) and (3.2.4), we get

$$\sum_{j=1}^{k} h_j(x) \leq \sum_{j=1}^{m} y_j h_j(\bar{x}) \tag{3.3.8}$$

Consequently (3.3.7) and (3.3.8) yields

$$\sum_{i=1}^{k} \lambda_i \left[f_i(x) - \nu_i g_i(x) \right] + \sum_{j=1}^{m} y_j h_j(x) < \sum_{i=1}^{k} \lambda_i \left[f_i(\bar{x}) - \nu_i g_i(\bar{x}) \right] + \sum_{j=1}^{m} y_j h_j(\bar{x})$$

This contradicts (3.3.6)

Hence $\bar{x}$ is an efficient solution for (MFP1).

Again suppose hypothesis (b) holds

From the relation (3.2.1), (3.3.3) and (3.2.4), we get

$$\sum_{j=1}^{m} y_j h_j(x) \leq \sum_{j=1}^{m} y_j h_j(\bar{x}) \Rightarrow H(x) \leq H(\bar{x})$$

$\Rightarrow k\left(x, \bar{x}\right)\psi\left\{H(x) - H\left(\bar{x}\right)\right\} \le 0.$

Hence the □-quasi invexity of H(x) with respect to □□□□□ implies

$$\eta\left(x, \bar{x}\right)^T \nabla H\left(\bar{x}\right) + \rho\left\|x - \bar{x}\right\|^2 \le 0 \Rightarrow \eta\left(x, \bar{x}\right)^T \nabla H\left(\bar{x}\right) \le 0 \qquad (3.3.9)$$

From (3.3.1), we get

$$\sum_{i=1}^{k}\lambda_i\left[\nabla f_i\left(\bar{x}\right) - \nu_i\left(\nabla g_i\left(\bar{x}\right)\right)\right] + \sum_{i=1}^{m} y_i \nabla h_i\left(\bar{x}\right) = 0$$

$$\Rightarrow \nabla\, Q\left(\bar{x}\right) + \nabla H\left(\bar{x}\right) = 0$$

$$\Rightarrow \eta\left(x, \bar{x}\right)^T\left[\nabla Q\left(\bar{x}\right) + \nabla H\left(\bar{x}\right)\right] = 0$$

$$\Rightarrow \eta\left(x, \bar{x}\right)^T \nabla Q\left(\bar{x}\right) + \eta\left(x, \bar{x}\right)^T \nabla H\left(\bar{x}\right) = 0 \qquad [3.3.10]$$

Using (3.3.9) in (3.3.10), we get $\eta\left(x, \bar{x}\right)^T \nabla Q(x) \ge 0$. For $\rho \in R_+$, we have

$$\eta\left(x, \bar{x}\right)^T \nabla Q(x) + \rho\left\|x - \bar{x}\right\|^2 \ge 0$$

Since Q(x) is □-pseudo invex wih respect to □□□□, we obtained $Q(x) - Q\left(\bar{x}\right) \ge 0.$

$$\Rightarrow \sum_{i=1}^{k}\lambda_i\left[f_i(x) - \nu_i g_i(x)\right] \ge \sum_{i=1}^{k}\lambda_i\left[f_i\left(\bar{x}\right) - \nu_i g_i\left(\bar{x}\right)\right] \qquad (3.3.11)$$

If $\bar{x}$ was not an efficient solution to (MFP1), then from (3.3.6), we have

$$\sum_{i=1}^{k}\lambda_i\left[f_i(x) - \nu_i g(x)\right] < \sum_{i=1}^{k}\lambda_i\left[f_i\left(\bar{x}\right) - \nu_i g_i\left(\bar{x}\right)\right]$$

This contradicts (3.3.11)

Therefore $\bar{x}$ is an efficient solution for (MFP1)

IV. MIXED DUALITY IN FRACTIONAL PROGRAMMING

Dual problem :

$$\left(\text{MFDO}\right) = \text{Maximize}\frac{f(u)}{g(u)} = \frac{f_{1i}(u')}{g_{1i}(u')} + \frac{f_{2i}\left(u^2\right)}{g_{2i}\left(u^2\right)}$$

$$\text{Maximize } F(u) = \frac{f(u)}{g(u)}$$

$$= F_1\left(u^1\right) + F_2\left(u^2\right)$$

$$= \frac{f_{1i}\left(\mu^1\right)}{g_{1i}\left(u^1\right)} + \frac{f_{2i}\left(u^2\right)}{g_{2i}\left(u^2\right)} \quad i = 1, \ldots, \ell$$

$$= \left(f_{1i}\left(u^1\right) - \nu_{1i}\left(g_{1i}\right)\left(u^1\right)\right) + \left(f_{2i}\left(u^2\right) - \nu_{2i} g_{2i}\left(u^2\right)\right)$$

(MFD_1) maximize $\left[\left(f_{11}\left(u^1\right) - \nu_{11} g_{11}\left(u^1\right)\right) + \left(f_{21}\left(u^2\right) - \nu_{21} g_{21}\left(u^2\right)\right)\right], \ldots.$

$$\left[\left(f_{1\ell}\left(u^1\right) - \nu_{1\ell} g_{1\ell}\left(u^1\right)\right) + \left(f_{2\ell}\left(u^2\right) - \nu_{2\ell} g_{2\ell}\left(u^2\right)\right)\right]$$

$$= \left(F_1(u) \ldots F_\ell(u)\right) = F(u)$$

(MFD_2) maximize $\lambda F(u),\ \lambda_i \in R_+,\ i = 1, \ldots \ell$

All with subject to same constraint.

$$\nabla\left[\lambda F_1\left(u^1\right) + y_{1j}^T h_{1j}\left(u^1\right)\right] = 0 \qquad (3.4.5)$$

and $\nabla\left[\lambda F_2\left(u^2\right) + y_{1J}^T h_{1j}\left(u^2\right)\right] = 0,$

$f_{1i}\left(u^1\right) + y_{1j}^T h_{1j}\left(u^1\right) - \nu_{1i} g_{1i}\left(u^1\right) \geq 0$ for i = 1, …., k,

$$\text{and } f_{2i}\left(u^2\right) + y_{2j}^T h_{2j}\left(u^2\right) - \nu_{2i} g_{2i}\left(u^2\right) \geq 0 \text{ for } i = 1, \ldots k \qquad (3.4.6)$$

$$y_{2j}^T h_{2j}\left(u^1\right) \geq 0, \quad y_{2j}^T h_{2j}\left(u^2\right) \geq 0 \quad y_{2j} \in R^{m-|j_1|} \qquad (3.4.7)$$

$$u^1 \geq 0 \ u^2 \geq 0\ ;\ \nu_{1i}, \nu_{2i} \geq 0 \qquad (3.4.8)$$

Theorem 4.1 (weak duality)

Let x be a feasible solution for the primal and $\left(u_0, y, \nu\right)$ be feasible for dual

If $F(u) = f_i(u) + y_{ij}^T h_{ij}(u) - \nu_i g_i(u),\ i = 1, i, \ldots k$ is ρ-pseudo invex with respect to η, ρ, for

$y_{2j} \in R^{m-|j_1|}, y_{2j}^T h_{2j}(u)$ is ρ -quasi invex with respect to η, ρ, then

$$\text{Inf}\left(\lambda F(x)\right) \geq \text{Sup}\left(\lambda F(u)\right)$$

Proof : Now from the primal and dual constraint, we have $h(x) \leq 0$ and $y_{2j}^T\ h_{2j}(u) \geq 0$

$$\text{So } y_{2j}^T\ h_{2j}\left(x^1\right) - y_{2j}^T h_{2j}\left(u^1\right) \leq 0 \quad \text{and } y_{2j}^T h_{2j}\left(x^2\right) - y_{2j}^T h_{2j}\left(u^2\right) \leq 0 \qquad (3.4.9)$$

Since $y_{2j}^T h_{2j}$ is ρ-quasi invex with respect to η and in view of (3.4.9) for $x, u \in R^n$, we have $\eta\left(x^1, u^1\right)^T \nabla\left[y_{2j}^T h_{2j}\left(u^1\right)\right] + \rho\left\|x^1 - u^1\right\|^2 \leq 0,\ \eta\left(x^2, u^2\right)^T \nabla\left[y_{2j}{}^T h_{2j}\left(u^2\right)\right] + \rho\left\|x^2 - u^2\right\| \leq 0$

$$\Rightarrow \eta\left(x^1, u^1\right)^T \nabla\left[y_{2j}^T\ h_{2j}\left(u^1\right)\right] \leq 0 \text{ and } \eta\left(x^2, u^2\right)^T \nabla\left[y_{2j}^T h_{2j}\left(u^2\right)\right] \leq 0 \qquad (3.4.10)$$

From the dual constraint (3.4.5), we have

$$\nabla\left[\lambda F_1\left(u^1\right)+y_{1j}^T h_{1j}\left(u^1\right)\right]=0 \text{ and } \nabla\left[\lambda F_2\left(u^2\right)+y_{2j}^T h_{2j}\left(u^2\right)\right]=0$$

Since $\eta\left(x^1,u^1\right)\in R^{|J_1|}$ and $\eta\left(x^2,u^2\right)\in R^{|J_2|}$, we have

$$\eta\left(x^1,u^1\right)^T\nabla\left[\lambda F_1\left(u^1\right)+y_{2J}^T h_{2j}\left(u^1\right)\right]=0 \text{ and } \eta\left(x^2,u^2\right)^T\nabla\left[\lambda F_2\left(u^2\right)+y_{2J}^T h_{2j}\left(u^2\right)\right]=0$$

$$\Rightarrow \eta\left(x^1,u^1\right)^T\nabla\left(\lambda F_1\left(u^1\right)\right)+\eta\left(x^1,u^1\right)^T\left(y_{2j}^T h_{2j}\left(u^1\right)\right)=0 \text{ and } \eta\left(x^2,u^2\right)^T\nabla\left(\lambda F_2\left(u^2\right)\right)+\eta\left(x^2,u^2\right)^T\left(y_{2j}^T h_{2j}\left(u^2\right)\right)=0$$

Using (3.4.10), we get

$$\eta\left(x^1,u^1\right)^T\nabla\left(\lambda F_1\left(u^1\right)\right)\geq 0$$

$$\Rightarrow \eta\left(x^1,u^1\right)^T\nabla\left[\sum_{i-1}^{k}\lambda_i\left\{f_i\left(u^1\right)+y_{1J}^T h_{1j}\left(u^1\right)-\nu_i g_i\left(u^1\right)\right\}\right]+\rho\left\|x^1-u^1\right\|^2\geq 0 \text{ and } \eta\left(x^2,u^2\right)^T\nabla\left(\lambda F_2\left(u^2\right)\right)\geq 0$$

$$\Rightarrow \eta\left(x^2,u^2\right)^T\nabla\left[\sum_{i=1}^{k}\lambda_i\left\{f_i\left(u^2\right)+y_{1J}^T h_{1j}\left(u^2\right)-\nu_i g_i\left(u^2\right)\right\}\right]+\rho\left\|x^2-u^2\right\|^2\geq 0 \qquad (3.4.11)$$

Since F(u) is ρ-pseudoinvex with respect to η and (3.4.11), we get

$$\sum_{i=1}^{k}\lambda_i\left[\left\{\left(f_{1i}\left(x^1\right)+y_{1j}{}^T h_{1j}\left(x^1\right)-\nu_{1i}g_{1i}\left(x^1\right)\right)+\left(f_{2i}\left(x^2\right)+y_{1i}^T h_{1j}\left(x^2\right)-\nu_{2i}g_{2i}\left(x^2\right)\right)\right\}\right]-$$

$$\left[\left\{\left(f_{1i}\left(u^1\right)+y_j^T h_{1j}\left(u^1\right)\right)-\nu_{1i}g_{1i}\left(u^1\right)+\left(f_{2i}\left(u^2\right)+y_{1j}^T h_{1j}\left(u^2\right)-\nu_{2i}g_{2i}\left(u^2\right)\right)\right\}\right]\geq 0$$

$$\Rightarrow \sum_{i=1}^{k}\lambda_i\left[\left\{f_{1i}\left(x^1\right)+y_{1i}^T h_{1j}\left(x^1\right)-\nu_{1i}g_{1i}\left(x^1\right)\right\}+\left\{f_{2i}\left(x^2\right)+y_{1j}^T h_{1j}\left(x^2\right)-\nu_{2i}g_{2i}\left(x^2\right)\right\}\right]\geq$$

$$\sum_{i=1}^{k}\lambda_i\left[\left\{f_{1i}\left(u^1\right)+y_{1i}^T h_{1j}\left(u^1\right)-\nu_{1i}g_{1i}\left(u^1\right)\right\}+\left\{f_{2i}\left(u^2\right)+y_{1j}^T h_{1j}\left(u^2\right)-\nu_{2i}g_{2i}\left(u^2\right)\right\}\right] \qquad (3.4.12)$$

Since $h(x)\leq 0 \Rightarrow y_{1j}^T h_{1j}(x)\leq 0$ for $y_{1j}\geq 0$

So (3.4.12) implies that

$$\sum_{i=1}^{k}\lambda_i\left[\left\{f_{1i}\left(x^1\right)-\nu_{1i}g_{1i}\left(x^1\right)\right\}+\left\{f_{2i}\left(x^2\right)-\nu_{2i}g_{2i}\left(x^2\right)\right\}\right]\geq\sum_{i=1}^{k}\lambda_i\left[\left\{f_{1i}\left(u^1\right)-\nu_{1i}g_{1i}\left(u^1\right)\right\}+\left\{f_{2i}\left(u^2\right)-\nu_{2i}g_{2i}\left(u^2\right)\right\}\right]$$

$$\Rightarrow \text{Inf}\left(\lambda F(x)\right)\geq \text{Sup}\left(\lambda F(u)\right)$$

Theorem 4.2 (Strong Duality) :

Let $\bar{x}$ be properly efficient solution of (MFPO) and a constraint qualification (Mangasarian [ ]) is satisfied. Then there exists a feasible solution $\left(\bar{u},\bar{y},\bar{v},\bar{w}\right)$ for dual and corresponding objective values are equal to zero. Furhter if $\left(\bar{u},\bar{y},\bar{v},\bar{w}\right)$ is feasible for dual, $F_i(u)$ is ρ-pseudo invex and $y_{j_2}^T h_{j_2}$ is η –quasi invex then $\left(\bar{x}=\bar{u},\bar{y},\bar{v},\bar{w}\right)$ is properly efficient for (MFDO).

Proof : Since $\bar{x}$ is a properly efficient solution of (MFPO), it is optional for (MFP_2). Then by lemma (3.3.3), we have

$$\nabla \lambda F_i(\bar{x}) + y_{J_2}^T \Delta h_{J_2}(\bar{x}) = 0$$

$$F_i(\bar{x}) = 0,$$

$$y_{J_2}^T \nabla h_{J_2}(\bar{x}) = 0, \quad y^T h(\bar{x}) = 0,$$

$$u \geq 0 \quad \nu_0 \geq 0$$

These are nothing but the dual constraints. So $(\bar{u}, \bar{y}, \bar{v}, \bar{w})$ is feasible for dual. So the objective values of (MFP_2) and (MFD_2) are equal to zero. It follows from theorem 3.4.2 and for any feasible solution $(\bar{u}, \bar{y}, \bar{v}, \bar{w})$ of dual $\lambda F(\bar{u}) \leq \lambda F(\bar{x})$. So $(\bar{x}, \bar{y}, \bar{v}, \bar{w})$ is optimal solution of (MFD_2). Then applying Lemma 3.3.1 and Lemma 3.3.2, we conclude that $(\bar{x}, \bar{y}, \bar{v}, \bar{w})$ is properly efficient for (MFD0).

V. CONCLUSION

In this paper, we introduced three approach given by Dinkelbaih [11], Jagannathan [12] and Yang at.al [10] for both primal and mixed type dual of a nondifferentiable multiobjective frictional programming problem. The results developed in this paper can be further extended to second order mixed type fractional programming problem and nondifferentiable fractional programming problems.

REFERENCES

[1] Schaible, S. (1995) : Fractional Programming, Handbook of Global Optimization, Edited by Horst, R. and Pardalos, P.M., Kluwer Academic Publishers, Dordrecht Netherlands 495-608.

[2] Bectar, C.R., Chandra, S. and Husain, I. (1993) : Optimality conditions and subdifferentiable multiobjective fractional programming, J. Optim. Theory Appl. 79, 105-125.

[3] Bectar, C.R., Chandra, S. and Abha, (2001) : On mixed symmetric duality in mathematical programming, J. Math, Anal, Appl. 259, 346-356.

[4] Xu, Z. (1996) : Mixed type duality in multiobjective programming problems, J. Math. Anal. Appl., 198, 621-635.

[5] X. M. Yang, X. Q. Yang and K. L. Teo (2003) : Nondifferentiable second order symmetric duality in mathematical programming with F-convexity, European Jour, of Operational Research 144, 554-559.

[6] X. M. Yang, K. L. Teo and X. Q. Yang (2003) : Mixed symmetric duality in nondifferentiable mathematical programming, Indian Journal of Pure and Applied Mathematics 34, 805-815.

[7] Hanson, M. A (1981) : On sufficiency of Kuhn-Tucker condition, J. Math. Anal. Appl., 80, 545-550.

[8] Mishra, S.K. (2000) : Second order symmetric duality in mathematical programming with F-convexity, European J. Oper. Res. 127, 507-518.

[9] Kim, D. S. and Song, Y. R. (1998) : Generalized symmetric duality for multiobjective nonlinear mixed integer ;programming, Nonlinear Funct, Anal. Appl., 3, 209-220.

[10] X. M. Yang, X. Q. Yang, K. L. Teo and S. H. Hou (2005) : Second order symmetric duality in non-differentiable multiobjective programming with F-convexity, European Jour. Of Operational Research 164, 406-416.

[11] Dinklebaih, W. (1967) : On nonlinear fractional programming, Management Sci., 13(7), 492-498.

[12] Jagannathan, R. (1973) : Duality for nonlinear fractional programs. Zeitschrift pur Oper. Res. Ser. A-B, 17, 1-3.