Effect of Yule's Model on Economic Design of X-Bar Control Chart for Independent Observations

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Abstract - In this paper we investigate the effect of autocorrelation on the economic design of $\overline{X}$ control chart. We make Yule's model by its three different roots, for various values of roots we find out the optimum values of sample size n and sampling intervals h.It may be inferred that Yule's model seriously affects the optimum value of the sample size and optimum sampling interval. It is necessary to point out that the observations from the population should be taken in to account while designing a control chart, as the optimum values of the control chart parameters are affected by the processing model.

Keywords: Economic Design Control Chart, Autocorrelation, Yule's model.

I. INTRODUCTION

Statistical Process Control (SPC) is defined as the use of statistical and engineering methods in measuring, monitoring, controlling, and improving quality. It ensures that the process operate at its full probable to produce conforming product. Under SPC, a process desired to produce as much conforming product as possible with the smallest amount possible waste. We may be confronted with an industrial situation where the nature of data occurs as independent. Thus, there is a need for a procedure which enables us to deal with the observations which are independent, for that we design and adopt economic design control charts and accordingly continue our search for the assignable causes of variation and for the optimum parameters.Duncan (1956) projected the first economic model for determining the sample size (*n*), the interval between successive subgroups (*h*), and the control limits of an $\overline{X}$ Chart that minimizes the average cost when a single out-of-control state (assignable cause) exists. Duncan's cost modal includes the cost of an out-of-control condition, the cost of false alarms, the cost of searching for an assignable cause, the cost of sampling, inspection, evaluations, and plotting. Considerable attention has been devoted to the optimal economic determination of these three parameters.The economic design of control charts is used to determine various design parameters thatminimize total economic costs. The effect of production lot size on the quality of the product may also be significant. If the manufacture process shifts to an out-of-control state at the beginning of the creation run, the entire lot will contain more faulty items. Hence, it is alarming step to reduce the production cycle to decrease the fraction of faulty items and, to improve output quality. On the other hand, reduction of the production cycle may result in an increase in costs due to frequent setups. A balance must be maintained so that the total cost is minimized. The production of quality goods depends upon the operating condition of the machine tools; however, the performance of machine tools depends upon the maintenance policy. It is assumed that the cost of maintaining the equipment increases with age, therefore, an age replacement strategy is needed to minimize the total cost of the system, which will simultaneously improve quality of the product and maintenance policy.Since Duncan's (1956, 1971) pioneering work. Earlier work in this area was summarized by Montgomery (1980, 1991), Vance (1983). Svoboda (1991), Ho and Case (1994), Goel*et al*. (1968), Knappenberger and Grandage (1969), and Gibra (1971), provided extensive reviews of the economic design of process control charts. The objective of the economic design of an X control chart is to determine the optimal design parameter values of the sample size n, the sampling (inspection) interval h, and the control limit coefficient k to minimize the expected cost per unit time of operation. Furthermore, Banerjee and Rahim (1987) treated the sample size n and the control limit coefficient k as constants. The questioning of interaction between quality and manufacturing operation has been addressed recently by Gershwin and Kim (2005), and Colledani (2008). Their studies are the first investigations of how quality considerations can modify the production control. Singh and Singh (2012) discuss the power of mean chart under Yule's model with known coefficient of variation. The design of control charts involves the selection of three parameters: sampling size (n), control frequency (h), and control limits (L) in order to detect earlier tools and processes shifts (Montgomery (2004)). Singh *et al*.(2012, 2013) discusses the problems on Control charts for mean under correlated data and Variables Sampling Plan For

Correlated Data, Manzoor and Singh (2015) makes use of Markoff's model on Economic design of X-bar control charts under independent observations. Thus, Economic design of control charts is a method which aims at determining these parameters of a control chart in optimizing a cost function of the process monitored. A breakthrough has been the generalization of all these models by Lorenzen and Vance (1986), it is nowadays a reference in economic design, as it can be easily implemented and adapted.

II. MATHEMATICAL MODEL FOR THE COST FUNCTION

Duncan (1956) obtained an approximate function for the average net income per hour of using the control chart for mean of normal variables as:

$$I = V_0 - \frac{\eta MB + (\alpha T/h) + \eta W}{1+\eta B} - \frac{b+cn}{h} \qquad (1.1)$$

The above model includes

(i) the cost of an out-of –control conditions

(ii) the cost of false alarms,

(iii) the cost of finding an assignable cause and

(iv) the cost of sampling inspection, evolution, and plotting.

NOTATIONS

V_0 = the average income per hour when process is in control and process average is μ ,

V_1 = the average income per hour when process is not in control and process average is $\mu' = \mu + \delta\sigma$,

$M = V_0 - V_1$

η = the average number of times the assignable cause occur within an interval of time,

$B = ah + Cn + D$

$a = \frac{1}{P} - \frac{1}{2} + \frac{\eta h}{12}$,

h = Sampling interval in hours

Cn = the time required to take and inspect a sample of size n .

D = average time taken to find the assignable cause after a point plotted on the chart falls outside the control limits,

P = Probability of detecting an assignable cause when it exists,

$$P = \int_{-\infty}^{\mu - k\sigma/\sqrt{n}} g(\bar{x}/\mu')d\,\bar{x} + \int_{\mu + k\sigma/\sqrt{n}}^{\infty} g(\bar{x}/\mu')d\,\bar{x}$$

$$\cong 1 - \Phi(k - \delta\sqrt{n}) \text{ for } \delta > 0$$

Where $g(\bar{x}/\mu')$ is the density function of $\bar{X}$ when the true mean μ and $\Phi(x)$ is the normal probability

α = probability of wrongly indicating the presence of assignable cause.

$$= \int_{\mu - k\sigma/\sqrt{n}}^{\mu + k\sigma/\sqrt{n}} g(\bar{x}/\mu)d\,\bar{x} = 2\Phi(-k) \qquad (1.2)$$

T = The cost per occasions of looking for an assignable cause when no assignable cause exists,

W = the average cost per occasion of finding the assignable cause when it exist,

b = per sample cost of sampling and plotting, that is independent of sample size,

and c= the cost per unit of measuring an item in a sample.

The average cost per hour involved for maintain the control chart is $\frac{(b+cn)}{h}$. The average net income per hour of the process under the examination of the control chart for mean can be rewritten as,

$I = V_0 - L$

Where

$$L = \frac{\eta MB + (\alpha T/h) + \eta W}{1+\eta B} + \frac{b+cn}{h} \qquad (1.3)$$

L can now be treated as the per hour cost due to the surveillance of the process under the control chart. The probability density function for Yule's model is represented by the first two terms of Edgewoth series and P and α' are determined from the sampling distribution of mean and are written as.

$$P = 1 - \Phi(\xi) \quad (1.4)$$

Where $\xi = (k - \delta\sqrt{n})$

III. DERIVATION FOR OPTIMUM VALUE OF SAMPLE SIZE AND SAMPLING INTERVAL

One can determine the optimum value of sample size n and sampling interval h either by maximizing the gain function I or by minimizing the cost function L with respect to n and h, we get

$$\frac{\partial L}{\partial n} = \frac{\left\{ \begin{array}{l} (1+\eta B)\left(\eta M \frac{\partial B}{\partial n} + \frac{T}{h}\frac{\partial \alpha}{\partial n}\right) \\ -\left(\eta MB + \frac{\alpha T}{h} + \eta W\right)\eta \frac{\partial B}{\partial n} \end{array} \right\}}{(1+\eta B)^2} + \frac{c}{h} = 0 \quad (2.1)$$

$$\frac{\partial L}{\partial h} = \frac{\left\{ \begin{array}{l} (1+\eta B)\left(\eta M \frac{\partial B}{\partial h} - \frac{\alpha T}{h^2}\right) \\ -\left(\eta MB + \frac{\alpha T}{h} + \eta W\right)\eta \frac{\partial B}{\partial n} \end{array} \right\}}{(1+\eta B)^2} - \left(\frac{b+cn}{h^2}\right) = 0 \quad (2.2)$$

Where,

$$\frac{\partial B}{\partial n} = \frac{-h}{P^2}\frac{\partial P}{\partial n} + c, \quad \frac{\partial L}{\partial h} = P^{-1} - \frac{1}{2} + \frac{\eta h}{6}, \quad \frac{\partial \alpha}{\partial n} = 0 \quad \text{and}$$

$$\frac{\partial P}{\partial n} = \frac{\delta}{2\sqrt{n}}\phi(\xi) \quad (2.3)$$

The solutions of the equations (2.1) and (2.2) for n and h are

$$\begin{array}{l} \eta h\left(M - \eta MB - \frac{\alpha T}{h} - \eta W\right)\frac{\partial B}{\partial n} + \frac{T}{h}\frac{\partial \alpha}{\partial n} \\ + \eta B\left(\eta M \frac{\partial B}{\partial n} + \frac{T}{h}\frac{\partial \alpha}{\partial n}\right) + c(1+\eta B)^2 = 0 \end{array} \quad (2.4)$$

$$\begin{array}{l} \eta h^2\left(M - \eta MB - \frac{\alpha T}{h} - \eta W\right)\frac{\partial B}{\partial h} \\ -\alpha T(1+\eta B) + \eta^2 h^2 MB\frac{\partial B}{\partial n} - (b+cn)(1+\eta B)^2 = 0 \end{array} \quad (2.5)$$

By assuming η small and note that the optimum h is approximately of order of $\frac{1}{\sqrt{\eta}}$, we neglect terms with ηB containing ηWc, $\frac{\alpha' T}{h}$ and the terms equating higher powers of η. The equations (2.4) and (2.5) are simplified and put in the following form

$$\frac{-\eta h^2 M}{P^2}\frac{\partial P}{\partial n} - \eta \alpha T + c = 0 \quad (2.6)$$

$$\eta M h^2\left(\frac{1}{P} - \frac{1}{2}\right) - (\alpha T + b + cn) = 0 \quad (2.7)$$

From the equation (2.7) we get

$$h = \left\{ \frac{\alpha T + b + cn}{\eta M\left(\frac{1}{P} - \frac{1}{2}\right)} \right\}^{\frac{1}{2}} \quad (2.8)$$

By eliminating h from the equation (2.6), we get,

$$-\frac{\alpha T + b + cn}{P^2\left(\frac{1}{P} - \frac{1}{2}\right)}.\frac{\partial P}{\partial n} - \eta \alpha T + c = 0 \quad (2.9)$$

The values of n for which the equation (2.9) satisfy yield us the required optimum value of sample size n. Substituting this value of n in equation (2.8), we find the optimum value of the sampling interval h.

IV. DERIVATION OF THE OPTIMUM VALUES OF SAMPLE SIZE AND SAMPLING INTERVAL UNDER UNDER YULE'S MODEL.

In industrial quality control a fundamental assumption is that the observations are uncorrelated, for that we consider a mechanized process where a quality characteristic is measured. This situation may occur in a discrete mechanized process which produces discrete time 1, 2, 3, ... , n with one quality characteristic of concern. The mechanized process may also be continuous and the quality characteristic of concern is measured at discrete equidistant points. Suppose the quality characteristic are x_1, x_2, x_3, ... ,x_n, for our purpose we consider a reasonable model known as Yule's Model as

$$x_t = \mu + \xi_t \quad (3.1)$$

Where, μ is constant and ξ_t is a wide sense stationary time series with mean zero and standard devotion σ. A Durban and Watson (1950) 'd' statistic can be used to detect the presence or absence of serial correlation test is confirmed. If the serial correlation exists we use identification technique to define the nature of ξ_t. Unfortunately, the independence assumption is often violated in many types of manufacturing process, the development of sensing and measurement technology has made it possible, in many causes, to measure critical dimensions on every unit produced. All manufacturing processes are driven by initial elements, and when the frequency of sampling becomes short relative to the process time constant the sequence of process observations will be auto correlated. Suppose that a correlation test revealed the presence of data dependence and the identification techniques suggest that an autoregressive process of order one. For the model (3.1), the second order dependence is expressed through

$\xi_t = \alpha_1\xi_{t-1} + \alpha_2\xi_{t-2} + \varepsilon_t$; t=1, 2, ... ,n (3.2) where, ξ_t has the usual properties:

(i) $\varepsilon_t \sim N(0,\sigma_\varepsilon^2)$, $Cov(\varepsilon_t,\varepsilon_\gamma) = \begin{cases}\sigma_\varepsilon^2, t=\tau \\ 0,\ t\neq\tau\end{cases}$ (3.3)

$E(\xi_t)=0$, Since $E(\varepsilon_t)=0$, for all t,

Furthermore the process variance is

$$V(x_t)=\sigma^2=\frac{1-\alpha_2}{(1+\alpha_2)}\left[\frac{\sigma_\varepsilon^2}{(1-\alpha_2)^2-\alpha_1^2}\right]$$

Following Kendall and Stuart (1976) it can be shown that for stationary, the roots of the characteristic equation of process in equation (3.2) $\phi(B)=1-\alpha_1 B-\alpha_2 B^2$ (3.4)

The unit circle, which implies that the parameters α_1 and α_2 must satisfy the following conditions

$\alpha_1+\alpha_2<1$, $\alpha_2-\alpha_1<1$ and $-1<\alpha_2<1$ (3.5)

Suppose that R_1^{-1} and R_2^{-1} are the roots of the characteristic equation of the process given by equation (3.4) then $R_1=\frac{\alpha_1+\sqrt{\alpha_1^2+4\alpha_2}}{2}$, $R_2=\frac{\alpha_1-\sqrt{\alpha_1^2+4\alpha_2}}{2}$

For stationary we require $|R_i|<1, i=1,2.$ thus, there occurs three situations

(i) Roots R_1 and R_2 are real and distinct $\alpha_1^2-4\alpha_2>0$

(ii) Roots R_1 and R_2 are real and equal $\alpha_1^2-4\alpha_2=0$

(iii) Roots R_1 and R_2 are complex conjugate $\alpha_1^2-4\alpha_2<0$

When serial correlation is present in the data, we have for the distribution of the sample mean $\overline{x}$, the mean and variance is given by

$$E(\overline{x})=\mu \text{ and } Var(\overline{x})=\frac{\sigma^2}{n}\lambda(\alpha_1,\alpha_2,n)=\frac{\sigma^2}{n}g_{AR(2)}^2$$

Where $\lambda(\alpha_1,\alpha_2,n)=g_{AR(2)}^2$ depends on the nature of the roots R_1 and R_2, and for different situations is given as follows

(1) R_1 and R_2 are real and distinct,

$$\lambda(\alpha_1,\alpha_2,n)=\begin{bmatrix}\frac{R_1(1-R_2^2)}{(R_1-R_2)(1+R_1R_2)}\lambda(R_1,n)\\ -\frac{R_2(1-R_1^2)}{(R_1-R_2)(1+R_1R_2)}\lambda(R_2,n)\end{bmatrix}$$
$$=\lambda_{rd}(\alpha_1,\alpha_2,n)$$

Where, $\lambda(R_2,n)=\left[\frac{1+R}{1-R}-\frac{2R}{n}\frac{(1-R^n)}{(1-R)^2}\right]$

(i) R_1 and R_2 are real and equal,

$$\lambda(\alpha_1,\alpha_2,n)=\left[\frac{1+R}{1-R}-\frac{2R}{n}\frac{(1-R^n)}{(1-R)^2}\right]\left[1+\left\{\frac{(1+R)^2(1-R^n)\\ -n(1-R^2)(1+R^n)}{(1-R^2)(1-R^n)}\right\}\right]$$
$$=\lambda_{re}(\alpha_1,\alpha_2,n)$$

(ii) R_1 and R_2 are complex conjugate

$$\lambda(\alpha_1,\alpha_2,n)=\left[\gamma(d,u)+\frac{2d}{n}(W(d,u,n)+z(d,u,n)\right]$$
$$=\lambda_{cc}(\alpha_1,\alpha_2,n)$$

Where, $\gamma(d,u)=\dfrac{1-d^4+2d(1-d^2)\cos u}{(1+d^2)(1+d^2-2d\cos u)}$

$$W(d,u,n)=\frac{2d(1+d^2)\sin u-(1+d^4)\sin 2u-d^{n+4}\sin(n-2)u}{(1+d^2)(1+d^2-2d\cos u)^2\sin u}$$

$$Z(d,u,n)=\frac{2d^{n+3}\sin(n-1)u-2d^{n+1}\sin(n+1)u-d^n\sin(n+2)u}{(1+d^2)(1+d^2-2d\cos u)^2\sin u}$$

$d^2=-\alpha_2$ and $u=\cos^{-1}\left(\dfrac{\alpha_1}{2d}\right)$

We suppose that the noise variance is known. The real valued parameters α_1 and α_2 (the autoregressive parameters) determines the influence of the preceding time point (t-1) and (t-2) on the present time t. We assume that the in control value $\alpha_1 = \alpha_2=0$ for the autoregressive parameters may shift to an out-of-control value $(\alpha_1,\alpha_2)\neq 0$. Further, the distribution of the sample average will have mean μ and standard deviation $\dfrac{\sigma^2}{n}\lambda(\alpha_1,\alpha_2,n)$ Therefore the probability density function under Yule's model for independent case is

$$P_{AR(2)}=1-\Phi(\xi_{AR(2)}) \tag{3.6}$$

$$\alpha_{AR(2)}=\alpha \text{, where } \xi=\frac{(k-\delta\sqrt{n})}{g_{AR(2)}} \tag{3.7}$$

$\alpha=2\Phi\left(\dfrac{-k}{g_{AR(2)}}\right)$, For Yule's model, the equation (2.1) and (2.2) will reduce in following form as

$$\frac{\partial L}{\partial n}=\frac{\left\{\begin{array}{l}(1+\lambda B)\left(\lambda M\dfrac{\partial B}{\partial n}+\dfrac{T}{h}\dfrac{\partial\alpha_{AR(2)}}{\partial n}\right)\\ -\left(\lambda MB+\dfrac{\alpha_{AR(2)}T}{h}+\lambda W\right)\lambda\dfrac{\partial B}{\partial n}\end{array}\right\}}{(1+\lambda B)^2}+\frac{c}{h}=0 \tag{3.8}$$

$$\frac{\partial L}{\partial h}=\frac{\left\{\begin{array}{l}(1+\lambda B)\left(\lambda M\dfrac{\partial B}{\partial h}-\dfrac{\alpha_{AR(2)}T}{h^2}\right)\\ -\left(\lambda MB+\dfrac{\alpha_{AR(2)}T}{h}+\lambda W\right)\lambda\dfrac{\partial B}{\partial h}\end{array}\right\}}{(1+\lambda B)^2}-\left(\frac{b+cn}{h^2}\right)=0 \tag{3.9}$$

Where,

$\dfrac{\partial B}{\partial n}=\dfrac{-h}{P^2_{AR(2)}}\dfrac{\partial p_{AR(2)}}{\partial n}+c$, $\dfrac{\partial B}{\partial h}=\dfrac{1}{p_{AR(1)}}-\dfrac{1}{2}+\dfrac{\lambda h}{6}$ and

$\dfrac{\partial\alpha}{\partial n}=0$

$$\frac{\partial P_{AR(2)}}{\partial n}=\frac{\delta}{2\sqrt{n}g_{AR(2)}}\phi(\xi_{AR(2)}) \tag{3.10}$$

By solving the equation (3.8) and (3.9) we get

$$h_{AR(2)}=h=\left\{\frac{\alpha_{AR(2)}T+b+cn}{\lambda M\left(\dfrac{1}{P_{AR(2)}}-\dfrac{1}{2}\right)}\right\}^{\frac{1}{2}} \tag{3.11}$$

and, $$-\frac{\alpha_{AR(2)}T+b+cn}{P^2_{AR(2)}\left(\dfrac{1}{P_{AR(2)}}-\dfrac{1}{2}\right)}\cdot\frac{\partial P_{AR(2)}}{\partial n}-\eta\alpha_{AR(2)}T+c=0 \tag{3.12}$$

The values of n for which the equation (3.12) satisfy yield us the required optimum value of sample size *n*. Substituting this value *n* in equation (3.11), we find the optimum value of the sampling interval *h* under Yule's model.

V. NUMERICAL ILLUSTRATION AND RESULT

For the purpose of numerical illustration, we take k=3, 2.5, 2, 1.5, 1 δ = 1.0, 1.5, and 2.0, η=0.01, M=100, W=25, T=50, c=0.05, D=2,b=0.5, we determine the optimum values of sample size n and sampling interval h which are presented in below table. Here mathematical investigation has been made to study the effect of Yule's model on the economic design of $\overline{X}$ - control chart for independent observation. We employ the economic design model of $\overline{X}$ - chart for Yule's shock model proposed by Edgeworth series for the production cycle time and cost parts.

Moreover, under Yule's model three different situations arises which are (i) Roots are real and distinct (ii) Roots are real and equal (iii) Roots are complex conjugate, the significant effects are seen on $\overline{X}$ control chart for the above three situation. The sample size required to detect given shift increases with the values of the roots when they are i) real and equal ii) real and distinct although the sampling interval and sample size is not much affected for independent and complex conjugate cases. Thus, there is a need for a procedure which enables us to deal with the data which are independent, to design control charts accordingly and to continue our search for the assignable causes of variation. It may be inferred that independent observations affects considerably the optimum value of the sample size and optimum sampling interval. It is necessary to point out that the errors of the population should be taken in to account while designing a control chart as the optimum values of the control chart parameters are affected by the independent observations under Yule's model.

Table: Optimum sample size (n) and sampling interval (h) under AR (2) model for independent case.

Independent case	k= 3		k= 2.5		k= 2		k= 1.5		k= 1	
δ↓	**n**	**h**	**n**	**h**	**n**	**h**	**n**	**h**	**n**	**h**
1	23	2.3371	20	2.4134	19	3.0258	35	4.6216	29	6.2076
1.5	11	1.8026	9	1.9963	9	2.7064	16	4.1978	14	5.9551
2	6	1.5648	6	1.8157	6	2.5740	9	4.0314	8	5.8593
Real and Equal case	k= 3		k= 2.5		k= 2		k= 1.5		k= 1	
δ↓	**n**	**h**	**n**	**h**	**n**	**h**	**n**	**h**	**n**	**h**
1	121	6.0858	158	7.1812	98	7.0731	99	8.0460	86	8.8742
1.5	50	4.6249	48	5.2287	44	6.0645	44	7.2525	41	8.2789
2	16	3.5081	26	4.5540	24	5.5046	26	6.8561	22	7.9320
Real and Distinct case	k= 3		k= 2.5		k= 2		k= 1.5		k= 1	
δ↓	**n**	**h**	**n**	**h**	**n**	**h**	**n**	**h**	**n**	**h**
1	322	10.3156	293	10.3342	256	10.3917	327	11.7390	370	12.4366
1.5	100	7.0270	91	7.2406	75	7.7402	110	9.3412	97	9.7355
2	40	5.0846	37	5.7809	36	6.6259	45	8.1316	38	8.7761
Complex Conjugate case	k= 3		k= 2.5		k= 2		k= 1.5		k= 1	
δ↓	**n**	**h**	**n**	**h**	**n**	**h**	**n**	**h**	**n**	**h**
1	23	2.3543	20	2.4698	20	3.1370	34	4.5952	36	6.3363
1.5	11	1.8865	10	2.2111	11	3.0444	18	4.3756	15	6.1149
2	7	1.7380	7	2.1765	7	3.1088	10	4.3768	10	6.1600

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