Optimization of Vendor-Inventory Model Under Simulation Approximation

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Abstract - The Optimization of a general vendor-inventory system under simulation approach methodology is presented. In the model, a problem related to replenishing inventories at retailers in distribution networks operated under the paradigm of vendor – managed inventory is analyzed. In this paper, we develop a simulation based optimization approach that can accommodate most of the complexity factors including several random variables and a multitude of the costs in the system.

Keywords - Supply chain, Simulation approach, Retailer networks, Vendor inventory.

I. INTRODUCTION

Vendor – Inventory systems are integral part of manufacturing industry. An efficient and effective simulation – based – optimization process for such inventory models becomes crucial for industries to maintain their manufacturing system, competitive edge especially in the face of suppliers and customers for inventory management. For many companies provided several scientific procedures for reducing inventory and optimizing service level in distribution networks of manufactured goods. There are numerous examples in the industry of organizations that have utilized efficient inventory – management system to reduce the total expenses incurred in Transportation, Storage and providing service. The application of these techniques has resulted in achieving significant cost reduction in producers of food products, chemical items and petroleum production

Vendor – managed inventory based systems face this problem on a daily or a weekly basis. In fact this study was motivated by a problem faced in a local industry. Solutions using the methods proposed in this paper can be obtained easily with computer programs, which can be run on personal computers and used directly in the decision making process. Since the simulation model is very general, the manager can easily change the system parameters for retailers, travel times and the demand rates as and when needed. A number of factors contribute randomness in the system. (i.e.) the customer arrival, the customer demand level and the randomness in the transportation.

Usually there are three sources of costs in these systems like transportation costs, inventory- holding costs and the stock-out costs. Modeling such a system mathematically is often challenging and difficult. As a result, we are developing a simulation based model that accommodates many features of a real-world system. The simulation model is combined with optimization techniques to generate optimal solutions.

The methodology presented here can accommodate a large number of features of real-world systems and can outperform two categories. One of the category outperformed is used in a local industry and the other one is derived from the news vendor model. The news vendor model is outperformed by simulation-based optimization. That implies that managers not interested in pursuing an elaborate simulation-optimization approach can resort to the simpler news vendor model for solution purposes. In addition we also prove that under the general assumptions made here, the cost function to be minimized is non-convex. Further, we develop a simulation-optimization methodology to solve the problem of determining the optimal quantities to be delivered each retailer.

Generally trucks are dispatched from the central warehouses to the retailers carrying the material needed. The problem considered in this paper is of tremendous relevance to the managers of warehouses who have to dispatch trucks with the right amounts of material.

In earlier researchers, Geon (seetlamma, 2001) obtained cost improvements by integrating its operations with those of its suppliers and customers for inventory management. Clark and Scarf (1960) and Eppen and Sehrage (1981) have formed the foundation of the underlying science in this field. Much of the existing literature is devoted to the development of mathematical models, and as such, it ignores the

Transportation costs, inventory-holding costs and stock-out costs. Mc Gavin etal (1993) ignored inventory-holding costs. Federgruen and Zipken (1984) developed a ‘myopic’ model which optimizes in the current time period but disregards costs in future time periods and Nahmias and smith (1994) developed models for the negative-binomial distribution. Many of the researchers, Minkoft (1993) Berman and Larson (2001) and Kleyuregt et al (2002) use stochastic dynamic programming.

II. MATHEMATICAL MODEL DESCRIPTION

The problem considered in this paper is to determine the optimal quantities to be delivered from the warehouse.

2.1 Problem Description

Our model considers the following costs to find the optimal solutions.

a. Inventory-holding costs
b. Stock-out costs
c. Transportation costs

The random variables governing our system are, the inter-arrival time of customers at each retailer, the quantity demanded by the customers, the service time for each truck and the travel time between the customer and the retailers and the same between the retailers. We have assumed that each retailer is distinct and has pivot values for the system parameters. Our objective is to minimize the average cost per unit time of operating the entire system.

2.2 Simulation-based Model Description

Assumptions & Notations

Assumptions

(i) Each retailer is not constant (distinct)
(ii) System parameters are unique values
(iii) Arrival rate of customers for the inventory-holding costs and stock-out costs are different for each retailer

Notations

q_i = the quantity to be delivered

i = particular retailer

n = number of retailers

$\overline{q}$ = ($q_{1,} q_2\ q_{3,\_\_\_\_\_\_\_\_,} q_n$) delivery quantities

$LS_i(\overline{q},t)$ = total number of Lost sales at time 't' in the simulation (time starts at 0) at the i^{th} retailer

$PI_i(\overline{q},t)$ = positive inventory at time t at the i^{th} retailer.

$[OC]_{ir}$ = operating cost per unit time

SO_l^i = the stock-out cost/ unit quantity of the sales lost at the i^{th} retailer

HC_s^i = the inventory-holding cost/quantity at the i^{th} retailer.

The problem is to determine the solution vector $\overline{q}$ in order to minimize the average cost/unit time.

(i.e.) $$AV(\overline{q}) = [OC]_{ir}\sum_{r=1}^{n} q_i + \sum_{\substack{i=1 \\ t\to\infty}}^{n} \frac{Lt[av]_i(\overline{q},t)}{t} \quad (1)$$

Where

$$[av]_i(\overline{q},t) = [SO]_l^i [LS]_i(\overline{q},t) + [HC]_s^i \int_0^t [PI](\overline{q},\tau)d\tau \quad (2)$$

Such that $q_i \geq 0$, for i = 1, 2, 3, ------ n

In equation (1), $[OC]_{ir}\sum_{r=1}^{n} q_i$, represents the transport costs, the second term denotes the expected cost of stock-outs and holding inventory on a unit time basis.

In equation (2) , the R.H.S denotes the costs due to stock-outs and the second term denotes the cost of holding inventory.

The First term in equation (2), $[SO]_l^i [LS]_i(\overline{q},t)$ is evaluated with a separate counter for the i^{th} retailer that is incremented wherever a lost sales occurs at the i^{th} retailer, while the second term is evaluated as follows.

(i.e.) $$[HC]_s^i \int_0^t [PI](\overline{q},\tau)d\tau = \sum_{j=0}^{\infty} J\delta_j(i,t)$$

Where $\delta_j(i,t)$ denotes the total duration of time interval starting from time 0 until the simulation clock strikes t, during which the i^{th} retailer has j units of positive inventory , $\delta_j(i,t)$ can be easily valuated in the simulation program.

Derivatives can be calculated numerically with a finite difference technique under the simulation perturbation methodology.

(i.e.) $$\overline{x} \leftarrow \overline{x} - \mu\nabla f(x)$$

Where

$$\nabla f(x) = \left\{\frac{\partial f(\overline{x})}{\partial x(1)}, \frac{\partial f(\overline{x})}{\partial x(2)}, -----\frac{\partial f(\overline{x})}{\partial x(n)}\right\}$$

In the finite difference method, the gradient is calculated numerically. By using the central difference formula we can easily obtain the gradient numerical results.

(i.e.) $$\frac{\partial f(w)}{\partial x} = \frac{f(a+h) - f(a-h)}{2h}$$

In the above, the simulator has to run twice for each decision variable in every iteration of the search algorithm – once to calculate $f(a+h)$ and once to calculate $f(a-h)$. Thus the simulator has to be run 2n times if n denotes the number of decision variables. Then as n increases the number of runs of the simulator also increases and consequently , the computational increases considerably. By simulation-based optimization, the algorithm is a stochastic search algorithm.

III. ORIGINAL SIMULATION ALGORITHM

Step I : Put K=1, in the solution vector in the K^{th} iteration. The process will stop when the step size μ becomes smaller than a pre-determined value $\mu_{\min}$. Then define a sequence $C^K = \frac{1}{K^{\xi}}$, We will fix ξ a value, $\xi \in (0,1)$

Define U and V such that , 0 < U < 1 , 0 < V < 1 and V < U

Step II : Compute the values of h(i) by using the following formula $h(i) = D(i)C^K$

Step III : Compute $f(\bar{x}^K + \bar{h})$ and $f(\bar{x}^K - \bar{h})$ by using the following process, where

(i.e.) $f(\bar{x}^K + \bar{h})$ =

$$f\left[\left(x^K(1) + h(1), x^K(2) + h(2) + ------ + x^K(n) + h(n)\right)\right]$$

$f(\bar{x}^K - \bar{h})$ =

$$f\left[\left(x^K(1) - h(1), x^K(2) - h(2), ------- x^K(n) - h(n)\right)\right]$$

Step IV : Consider the partial derivatives

$$\frac{\partial f(\bar{x}^K)}{\partial x^K(i)} = \frac{f(\bar{x}^K + \bar{h}) - f(\bar{x}^K - \bar{h})}{2h(i)}$$, by putting i= 1, 2, -----------n.

Step V : Compute $\bar{y}$ using the following method

$$\bar{y} \leftarrow x^K(i) - \mu \frac{\partial f(\bar{x}^K)}{\partial x^K(i)}$$, for i= 1,2,----------n.

Step VI : If $f(\bar{y}) < f(\bar{x}^K)$, then construct $\bar{x}^{K+1} = \bar{y}$, and $\mu \leftarrow \mu R_1$, otherwise go to Step I

Step VII : Increase K by 1, then form $\bar{x}^{K+1} = \bar{x}^K$ and extend μ from $\mu \leftarrow \mu R_2$

Step VIII : If $\mu \le \mu_{\min}$, then stop the procedure. Otherwise continue from step II.

From the above procedure, we used the simulation based optimization which response surfaces on smaller versions of the problem with retailers. We generated the cost of running the system (for a given set of parameter) using the neural network.

The values of the coefficient of determinant are provided in the following tables.

Table :1 The values of parameters used for each system.

System	C_S^1	C_S^2	C_l^1	C_l^2	λ_1	λ_2
1	0.02	0.008	4.0	4.2	120	80
2	0.02	0.008	2.5	2.3	60	65
3	0.02	0.008	3.0	2.8	120	80
4	0.02	0.008	3.0	2.8	60	65

Table :2 Coefficient of Determination

System	Coefficient of Determination
1	0.882
2	0.886
3	0.887
4	0.875

A 5- retailer network

In this section we describe our computational results for a more realistic 5-retailer network

Table 3 - Level definitions

System	Retailer 1	Retailer 2	Retailer 3	Retailer 4	Retailer 5
$C_S - 1$	0.02	0.008	0.0095	0. 023	0.025
$C_S - 2$	0.025	0.0225	0.036	0.033	0.034
$Cs - 3$	0.05	0.059	0.058	0.066	0.064
$C_I - 1$	2.5	2.4	2.45	2.52	2.56
$C_I - 2$	3	2.9	2.95	3.13	3.2
$C_I - 3$	4	4.1	3.95	4.2	4.16
λ -1	60	70	80	85	90
λ -2	75	85	60	72	102

From the above table we were able to do the performance of simulation perturbation method by using a reactive step size and it is returned to the original solution when the algorithm strikes unmatch solution. The rate is restored to its original value only after an improved solution is found.

IV. SENSITIVITY ANALYSIS

A full factorial experiment was designed to determine which factors affect the optimal objective function obtained from our Simulation-optimization approach. We have studied the following three factors.

a. The inventory-holding costs
b. The stock-out costs
c. The rate of arrival of customers at two levels.

V. CONCLUSIONS

The solution methodology presented here should find ready acceptance in the industry. Furthermore the simulation-optimization approach can be run on any personal computer. Determining the optimal quantities to be dispatched to each retailer from the ware house is a long-standing problem in the industry.

The solutions developed by the simulation optimization approach can be directly incorporated into the decision making technology of Warehouse and multi-godown managers.

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