Application Concept Solution of Fuzzy Goal Matrix Game

G.Sasikala, Dr. S. Kumara Ghuru

Department of Mathematics, Chikkanna Govt. Arts College, Tirupur, Tamilnadu

***Abstract* - The basic aim of this paper is to an application concept of solutions of two person zero sum games with Fuzzy pay offs and fuzzy goals. We assume that each player has a fuzzy goal for each of the pay offs. A degree of attainment of the fuzzy goal is defined and the max-min strategy with respect to the degree of attainment of the fuzzy goal is examined. If all the membership functions both for the fuzzy pay offs and for the fuzzy goals are linear the max-min solution is formulated as a non–linear programming problem and it is converted to a linear programming problem.**

Key Words: Fuzzy pay off matrix, fuzzy goal, max-min strategy, two-person sum games.

I. INTRODUCTION

Game theory is the study of the ways in which strategic interactions among rational players produces outcomes with respect to the performance utilities of those players, none of which might have been intended by any of them. A two person game where the players are defined as decision makers is a simplest care of game theory. Here we consider a two person zero sum game with fuzzy pay offs and fuzzy goals. A pay off matrix with elements represented by as fuzzy pay off matrix. The research on fuzzy games has been develop by Aubin[2] and Butnaria[3], Recently, Campos[4] has explored zero-sum fuzzy matrix game. The problem treated by compos was a game with a single pay off and the min-max problem was formulated using the fuzzy mathematical programming method.

In this paper a new application concept of solution of two-person zero-sum fuzzy matrix game is proposed. A fuzzy expected pay off is defined and a degree of attainment of a fuzzy goal is considered in games with fuzzy pay off matrices. The max-min solution with respect to a degree of attainment of a fuzzy goal is also defined.

The method for computing the solution of a two-person zero-sum game is proposed when membership function of fuzzy goals and shape functions of L-R fuzzy numbers for fuzzy pay off are linear. The max-min solution is formulated as a non linear programming then it is transformed as a linear programming problem.

II. BASIC DEFINITIONS

2.1. Definition: Zero-Sum Game with Fuzzy pay off

When player I chooses a pure strategy i ϵ I and player II chooses a pure strategy j ϵ J. Let $\overline{a_{ij}}$ be the fuzzy pay off for player I and - $\overline{a_{ij}}$ be a fuzzy pay off for player II. The fuzzy pay off $\overline{a_{ij}}$ represented by the L-R fuzzy number.

$$\overline{a_{ij}} = \left(a_{ij}^{1}, a_{ij}^{11}, a_{ij}^{111}\right)$$

Where a_{ij}^{l} *is a mean value*

a_{ij}^{ll} *is a left spread and*

a_{ij}^{lll} *is a right spread*

The two- person zero-sum fuzzy game can be represented as a fuzzy pay off matrix.

$$A_1 = \begin{pmatrix} \overline{a_{11}} & \overline{a_{12}} & \dots\dots\dots & \overline{a_{1n}} \\ \overline{a_{21}} & \overline{a_{22}} & \dots\dots\dots & \overline{a_{2n}} \\ \dots. & \dots. & \dots\dots\dots & \overline{a_{ij}} \\ \overline{a_{m1}} & \overline{a_{m2}} & \dots\dots\dots & \overline{a_{mn}} \end{pmatrix}$$

2.2 Definition: Fuzzy Expected Pay Offs

For any pair of mixed strategies x ε X, yε Y, fuzzy expected pay off of player I is defined as the fuzzy number.

$x A_1 y$ =

$$\left(\sum_{i=0}^{m}\sum_{j=0}^{n} a_{ij}^{1} X_i Y_j \;,\; \sum_{i=0}^{m}\sum_{j=0}^{n} a_{ij}^{11} X_i Y_j \;,\sum_{i=0}^{m}\sum_{j=0}^{n} a_{ij}^{111} X_i Y_j\right) \;\dots\dots(3)$$

Characterized by the membership function

$$\mu_{xA1y} : D \rightarrow [0,1] \qquad \dots..(4)$$

Where D is the domain of pay off for player.

2.3 Definition: Fuzzy-Goal

Let the domain pay off for player I be denoted by D∈ Π . Then the fuzzy $\bar{G}$ with respect to the payoff for player I is defined as the fuzzy set on the set D characterized by the membership function.

$$\mu_{ij} : D \rightarrow [0,1] \qquad \dots..(5)$$

2.4 Definition: Degree of Attainment

For any pair of mixed strategies (x,y), Fuzzy expected pay off for player I be denoted by xA_1y and set the fuzzy goal for player I denoted by $\bar{G}$. Then a fuzzy set expressing an attainment state of the fuzzy goal is represented by the intersection of the fuzzy expected pay off xA_1y and the fuzzy goal G. The membership function of the fuzzy set is represented as

$$\mu_{ij}\ (x,y)(\rho) = min\ [\mu_{xA1y}(\rho),\ \mu_{\tilde{G}}(\rho)]\ .....(6)$$

Where ρ ϵ D is a payoff for the player I.

A degree of the attainment of the fuzzy goal is defined as the maximum of the membership function by (6). We assume that player I suppose that player II choose a strategy $\bar{y}$ so as to minimize players I degree of attainment of the aggregated fuzzy goal.

That is Player I's degree of attainment of the aggregated Fuzzy goal, assuming to choose $\bar{x}$,will be

$$v(x) = min\ \mu_{ij}\ (x,y)(\rho^*)\ .....(7)$$

Hence, player I chooses a strategy so as to maximize his degree of attainment of the aggregated Fuzzy goal v(x).

2.5. Definition:

For any pair of mixed strategies (x,y), let a degree of attainment of the aggregated fuzzy goal for player I be denoted as

$$\bar{\mu}_{ij}\ (x,y)(\rho^*)$$

Then the player I's max-min value with respect to a degree of attainment of the aggregated fuzzy goal is denoted by

$$\max_{x \mathcal{E} X}\ \min_{y \mathcal{E} Y} \bar{\mu}_{ij}\ (x,y)(\rho)\ .....(8)$$

And such a strategy x is called the max-min solution with respect to the degree of attainment of the aggregated fuzzy goal.

III. SOLUTION PROCEDURE:

We show an application concept solution of two-person zero-sum game. We assume that membership functions of the fuzzy goals and shape functions of the fuzzy goals and shape functions of fuzzy numbers representing the fuzzy pay offs are linear.

A membership function of the player I's fuzzy goal is represented as

$$\mu_{\tilde{G}}\quad(\rho) = \begin{cases} 0 & , \quad if\ \ \rho < a_1 \\ \frac{\rho - a_1}{a_2 - a_1} & , \quad if\ a_1 < \rho < a_2 \\ 1 & , \quad if\ \ a_2 < \rho \end{cases} \quad .....(9)$$

Let a shape function for fuzzy numbers be

$$S(\rho) = T(\rho) = max((0,1) - |\rho|) \quad .....(10)$$

When player I and II choose pure strategies i∈ I and j∈ J, a pay off player I is represented as the fuzzy number

$$\overline{a_{ij}} = \left(a_{ij}^1,\ a_{ij}^{11}, a_{ij}^{111}\right)$$

Characterized by the membership function

$$\mu_{a_{ij}}(\rho) = \begin{cases} 0 & ; \quad if\ \rho < a_{ij}^1 - a_{ij}^{11} \\ \frac{\rho - a_{ij}^1 + a_{ij}^{11}}{a_{ij}^{11}} & ; \quad if\ a_{ij}^1 - a_{ij}^{11} \le \rho < a_{ij}^1 \\ \frac{a_{ij}^1 + a_{ij}^{11} - \rho}{a_{ij}^1} & ; \quad if\ a_{ij}^1 \le \rho \le a_{ij}^1 + a_{ij}^{11} \\ 0 & ; \quad if\ a_{ij}^{11} < \rho \end{cases} \quad ....(11)$$

Theorem:

For single-objective two-person zero-sum games, if membership functions of the fuzzy goal and shape functions of L-R fuzzy numbers for fuzzy payoff are linear. Players I's max min solution with respect to a degree of attainment of the aggregated fuzzy goal is identical to an optimal solution to the non-linear programming problem.

IV. SOLUTION PROCEDURE

Step:1

Identify a fuzzy goal for a pay off choose an initial point $y^1 \in$ Y and let i=1. Then formulate a relaxed problem of the equation and by taking I , points y^I , I= 1,2,3,…….I

Satisfying $y^I \in$ Y, $\sum_{j=1}^{n} y_j^I = 1$

Maximize σ ,

Subject to $\frac{\sum_{i=1}^{m}\sum_{j=1}^{n}\left(a_{ij}^1 + a_{ij}^{11}\right)x_i y_j^I - a_1}{\sum_{i=1}^{m}\sum_{j=1}^{n} a_{ij}^1 x_i y_j^I + a_1 - a_2} \ge \sigma$

$$\sum_{i=1}^{m} x_i = 1$$

And $x_i \ge 0$, i= 1,2,3,………….m.

Which is a linear fractional programming problem.

Step:2

Formulate the constraints

$$\sum_{i=1}^{m}\sum_{j=1}^{n}\left(a_{ij}^1 + a_{ij}^{11}\right)x_i y_j^I - a_1 \ge$$

$$\bar{\sigma}\ \left(\sum_{i=1}^{m}\sum_{j=1}^{n} a_{ij}^1\ x_i y_j^I + a_2 - a_1\right),\ \ i = 1,2,3\ \ ....I$$

$$\sum_{i=1}^{m} x_i = 1$$

And $x_i \ge 0$, i= 1,2,3, ….m.

By setting σ = $\bar{\sigma}$ in the constraints of the relaxed problem by equation (6).Compute as optimal solution (x*,σ*).

Step:3

Formulate the minimization linear programming problem

Minimize $\sum_{i=1}^{n}\sum_{j=1}^{n}\left(a_{ij}^1 + a_{ij}^{11}\right)x_i^1 z_j - a_{ij}^1$

Subject to $\sum_{j=1}^{n} z_j = t$

$$\sum_{i=1}^{m}\sum_{j=1}^{n} a_{ij}^1 x_i^1 z_j + (a2 - a1)t = 1\ ......(12)$$

$z_j \geq 0$, j = 1, 2 ……….n.

Step:4

Solve the problem (12) and obtain an optimal solution (z*,t*). Let the objective function value be denoted by φ(z*,t*).

Step: 5

If φ(z*,t*) ≥ σ* + λ terminate, λ where is a predetermined constant. Then X^1 is a max-min solution with respect to a degree of attainment of the fuzzy goal.

Otherwise, if φ (z*,t*) < σ* + λ respect the step2 to till the optimum solution obtained.

V. NUMERICAL EXAMPLE:

Consider a numerical example based on Cooks [5]. Assuming the each player has three pure strategies. We consider two-person Zero-Sum game with fuzzy pay offs be represented by

$$\bar{A} = \begin{bmatrix} (3,0.3,\ 0.3) & (6,\ 0.6,\ 0.6) & (2,\ 0.8,\ 0.8) \\ (-2,0.8,0.8) & (-3\ ,0.5\ ,0.5) & (8,0.2,\ 0.2) \\ (0\ ,0.2\ ,0.2) & (4\ ,\ 0.5\ ,0.5) & (-2\ ,0.8,0.8) \end{bmatrix}$$

let fuzzy goal $\bar{G}$ of player I for the single objective be representated by the following linear membership function

$$\mu_{\bar{G}}(\rho) = \begin{cases} 0 & ; \quad \rho' < -1 \\ \frac{\rho' + 1}{7.5} & ;\ if\ -1 \leq \rho' \leq 7.5 \\ 1 & ;\ if\ \ 7.5 < \rho' \end{cases}$$

We computed the maximum solution by Algorithm and obtained the following values.

$$x1 = 0.9837 \qquad x2 = 0.0816 \qquad x3 = 0$$
$$y1 = 0 \qquad y2 = 0 \qquad y3 = 1$$

The degree of attainment of the fuzzy goal for the max min solution.

VI. CONCLUTION

In this paper, application concepts of new solution technique considered the max-min solutions with respect to a degree of attainment of the fuzzy goal and have provided the computational method for their solutions. We have used Sakawa's method, the variable transformation by charnes and cooper (6) and the relaxation procedure for minimax problems for two-person zero-sum games. The membership functions of the fuzzy goal and shape functions of L-R type fuzzy numbers for fuzzy pay off are linear the maxmin solution with respect to a degree of presented with a new solution concept.

VII. FUTURE EXTENSION WORK

The above proposed model may be extended to investigate the application concept of this solution technique applicable m x n person matrix game. The membership functions of the fuzzy goal and shape functions of L – R type fuzzy numbers for the fuzzy pay offs are linearly independent with respect to degree of attainment under maxmin solution.

VIII. REFERENCES

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