Study of Fixed Points in Complete Metric Space by Symmetric Rational Expression

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Abstract: In this paper we introduce the different method to solve and prove some fixed point and common fixed pointtheorems for banach contraction in complete metric space by using new type rational contraction conditions. Our results extend and improve the recent ones.

Keywords: Fixed point, Common Fixed point, Continuous mapping, Complete Metric Space,cauchy sequence. AMS Classification: 45H10.

I. INTRODUCTION AND PRELIMINARIES:

Impact of fixed point theory in different branches of mathematics and its applications is immense. The first result on fixed points for contractive type mapping was the much celebrated Banach's contraction principle by S. Banach [1] in (1922). In the general setting of complete metric space, The important part of fixed point theory is Metric fixed point theory, because of its applications in different areas like variation and linear inequalities, improvement and approximation theory.Kannan[2]established fixed theorems in different spaces like Metric space. Rhoades (2001) and Khan et.al.(1984) author have proved that some unique fixed point theorems. In (1980) Jaggi[5] and Das [6] obtained some fixed point theorems with the mapping satisfying. Before going to prove our results first we define some important definitions.

Definition 2.1: Let (X, d) be a complete metric space, c ∈(0,1) and f: X → X be a mapping such that for each x,y∈X, d(T_x, T_y) ≤ cd (x, y) [1], Then f has a unique fixed point, a ∈ X,such that for each x ∈ X ,$\lim_{n\to\infty} f_{n_x} = a$. after the classical result, Kannan [2] gave a subsequently new contractive mapping to prove the fixed point theorem. Since then a number of mathematicians have been worked on fixed point theory dealing with mappings satisfying various type of contractive conditions. In (2002)A. Branciari [3] analyzed the existence of fixed point.

Definition 2.2: If T is self mapping of a complete metric space X into itself satisfying; d(T_x, T_y) $\leq \alpha$ [d(T_x, x)+ d(T_y,x)] for all x,y $\epsilon X$and 0 $\leq\alpha \leq 1$,then T has unique fixed point in X.See[2]

Definition 2.3:d(T_x, T_y) ≤ [d(x, T_y)+ d(y, T_x)] for allx,y ϵX and $0 \leq \alpha \leq 1$,then T has unique fixed point in X.See[4], A similar conclusion was also obtained by Chaterjee [7].

Definition 2.4: Let the rational expression

$$d(T_x, T_y) \leq \alpha \frac{d(x, T_x)\, d(y, T_y)}{d(x,y)} + \beta\, d(x,y)$$

for allx ,y ϵX, x ≠y and $0 \leq \alpha + \beta \leq 1$

thenThas unique fixed point in X. See[5]

Definition 2.5: let some fixed point theorems with the mapping satisfying:

$$d(T_x, T_y) \leq \alpha \frac{d(x, T_x) d(y, T_y)}{d(x,y) + d(x, T_y) + d(y, T_x)} + \beta\, d(x,y)$$

for allx ,y ϵX , x ≠ y,and$0 \leq \alpha + \beta \leq 1$ then T has unique fixed point in X. See[5][6].

Definition 2.6: A sequence { x_n} in metric space (X,d) is called Cauchy sequence if for

Each$\varepsilon < 0$, there exists a positive integern_0 such that$m, n \geq n_0 \Rightarrow$ d(x_m, x_n)<ε

Definition 2.7 : A complete metric space is a metric space in which every Cauchy sequence is convergent.

Definition 2.8 : A sequence { x_n} converges to x if$\lim_{n\to\infty} d(x_n, x) = 0$ in this case x is called a limit of $\{x_n\}$ and we write$x_n \to x$.

II. MAIN RESULTS

Theorem-3.1: Let T be a continuous self map, defined on a complete metric space X, satisfies the following condition ;

$$d(x_{n+1}, x_n) \leq \alpha \left[\frac{d^3(x,T_x) + d^3(x,T_y)}{1 + d^2(x,T_x)}\right] + \beta\left[d(y,T_y).\, d(y,T_x)\right] + \gamma\, d(x,y) + \eta[d(x,T_x) + d(y,T_y)] + \delta[d(x,T_y) + d(y,T_x)] \quad (3.1.1)$$

for all x,y,$x \neq y$and for some α,β,γ,η,δ ∈ [0,1) with (α+2β+2γ+2δ+η)<1.then T has fixed point in X.

Proof:Let $x_0$be an arbitrary point in X and we define a sequence $\{x_n\}$by means of iterates of T by setting, $T_{x_0}^n = x_n$ where n is a positive integer. If $x_n = x_{n+1}$ for some n,then x_n is affixed point of T.Taking $x_n \neq x_{n+1}$,for all n.

d(x_{n+1},x_n) = d$(T_{x_n},T_{x_{n-1}})$

$$d(x_{n+1},x_n) \le \alpha\left[\frac{d^3(x_n,T_{x_n})+d^3(x_n,T_{x_{n-1}})}{1+d^2(x_n,T_{x_n})}\right] + \beta\,[d(x_{n-1},T_{x_{n-1}}).\,d(x_{n-1},T_{x_n})] + \gamma\, d(x_n,x_{n-1})$$

$$+\eta[d(x_n,T_{x_n})+d(x_{n-1},T_{x_{n-1}})] + \delta\,[d(x_n,T_{x_n-1})+d(x_{n-1},T_{x_n})]$$

$$\le \alpha\left[\frac{d^3(x_n,x_{n+1})+d^3(x_n,x_n)}{1+d^2(x_n,x_{n+1})}\right] + \beta\,[d(x_{n-1},x_n).\,d(x_{n-1},x_{n+1})] + \gamma\, d(x_n,x_{n-1})$$

$$+\eta[d(x_n,x_{n+1})+d(x_{n-1},x_n)] + \delta\,[d(x_n,x_n)+\ d(x_{n-1},x_{n+1})]$$

$$\le \alpha\left[\frac{d(x_n,x_{n+1})+d(x_n,x_n)\{d^2(x_n,x_{n+1})+d^2(x_n,x_n)+d(x_n,x_{n+1})d(x_n,x_n)\}}{1+d^2(x_n,x_{n+1})}\right]$$

$$+\ \beta\, d(x_n,x_{n+1})\ +\gamma\, d(x_n,x_{n-1}) + \eta[d(x_n,x_{n+1})+d(x_{n-1},x_n)] + \delta d(x_{n-1},x_{n+1})$$

$$\le \alpha\left[\frac{d(x_n,x_{n+1})d^2(x_n,x_{n+1})}{d^2(x_n,x_{n+1})}\right] + \beta[\ d(x_n,x_{n+1})] + \gamma\, d(x_n,x_{n-1}) + \eta\ \ [d(x_n,x_{n+1})+d(x_{n-1},x_n)]$$

$$+\ \delta\,[d(x_{n-1},x_{n+1})]$$

$$d(x_n,x_{n+1}) \le \alpha d(x_n,x_{n+1}) + \beta d(x_n,x_{n+1}) + \gamma\, d(x_n,x_{n-1}) + \eta[d(x_n,x_{n+1})+d(x_{n-1},x_n)]$$

$$+\ \delta d(x_{n-1},x_{n+1}) \qquad (3.1.2)$$

$$d(x_n,x_{n+1}) \le (\alpha+\beta+\eta+\ \delta)d(x_n,x_{n+1}) + (\ \gamma+\eta+\delta)\ d(x_n,x_{n-1})$$

$$\{1-(\ \alpha+\beta+\eta+\ \delta)\}\ d(x_n,x_{n+1}) \le (\ \gamma+\eta+\delta)\ d(x_n,x_{n-1})$$

$$d(x_n,x_{n+1}) \le \frac{(\ \gamma+\ \eta+\delta)}{1-(\ \alpha+\beta+\eta+\ \delta)}\, d(x_n,x_{n-1})$$

$$\vdots$$

$$\left[\frac{(\ \gamma+\ \eta+\delta)}{1-(\ \alpha+\beta+\eta+\ \delta)}\right]^{ntimes}\ d(x_n,x_{n-1})$$

By the triangular inequality, we have m>n,

$$d(x_n,x_m) \le d(x_n,x_{n+1}) + d(x_{n+1},x_{n+2}) + + - - - d(x_{m-1},x_n)$$

$$\le (K^n + K^{n+1} + K^{n+2} + + + - - K^{m-1})\ d(x_1,x_0)$$

Where K = $\frac{(\gamma+\eta+\delta)}{1-(\alpha+\beta+\eta+\delta)} < 1$ (3.1.3)

$1-(\alpha+\beta+\eta+\ \delta) \le (\ \gamma+\eta+\delta)$ (3.1.4)

$(\alpha+\beta+2\eta+2\ \delta+\gamma) \le 1$ (3.1.5)

$\therefore\ d(x_n,x_m) \le \frac{K^n}{1-K} d(x_0,T_{x_0})$ (3.1.6)

So$\{x_n\}$is Cauchy sequence in X,So by completeness of X,there is apoint uϵX,such that $x_n \to u\ as\ n \to \infty$

Further,the continuity of T in X,implies

$$T(u) = \lim_{n\to\infty} Tx_n$$

$$= \lim_{n\to\infty} T_{x_n}$$

$$= \lim_{n\to\infty} x_{n+1}$$

$$= u$$

$\therefore$ u is fixed point of T in X.

Theorem3.2: Let T be the self map defined on a complete metric space (X,d) such that(3.1.1)holds.if for some positive integer p,T^p is continuous ,then T has a fixed point not unique.

Proof: We define a sequence $\{x_n\}$ as in Theorem-3.1 ,clearly it converges to some point u is (X,d). therefore its subsequence $\{x_{n_k}\}$,($n_k = k_p$) also converges to u.

$$T_u^p = T^p(\lim_{k\to\infty} x_{n_k}) = \lim_{k\to\infty} T^p_{x_{n_k}}$$

$$= \lim_{k\to\infty}(T_{x_{k+1}})$$

$$= u$$

Therefore u is a fixed point of T^p. Now ,we show that ,$T_u = u$.Let m be the smallest positive integer,s.t.$T_u^m = u, but T_u^q \neq u$,for q=1,2,3,-----m-1.

If m > 1,then by (3.1.1)

d(T_u, u) = d(T_u, T_u^m) = d[$T_u, T(T_u^{m-1})$]

$$d(u,T_u) \le \alpha\left[\frac{d^3(u,T_u)+d^3(u,T(T_u^{m-1}))}{1+d^2(u,T_u)}\right] + \beta\left[d\left(T_u^{m-1},T(T_u^{m-1})\right).d(T_u^{m-1},T_u)\right]$$

$$+ \gamma\, d(u,T_u^{m-1}) + \eta\,[d(u,T_u) + \left(d\left(T_u^{m-1},T(T_u^{m-1})\right)\right)]$$

$$+ \delta\,[d\left(u,T(T_u^{m-1})\right) + d(T_u^{m-1},T_u)] \qquad (3.2.1)$$

$$\le \alpha\left[\frac{d(u,T_u) + d\left(u,T(T_u^{m-1})\right)\{d^2(u,T_u) + d^2\left(u,T(T_u^{m-1})\right) + d(u,T_u).d\left(u,T(T_u^{m-1})\right)\}}{1+d^2(u,T_u)}\right]$$

$$+\beta[d(T_u^{m-1},u).d(T_u^{m-1},T_u)] + \gamma d(u,T_u^{m-1}) + \eta\,[d(u,T_u) + d(T_u^{m-1},u)]$$

$$+\delta[d(u,u) + d(T_u^{m-1},T_u)]$$

$$\le \alpha\left[\frac{\{d(u,T_u)+d(u\,u)\}\{d^2(u,T_u)+d^2(u,u)+d(u,T_u).d(u,u)\}}{1+d^2(u,T_u)}\right] + \beta[d(T_u^{m-1},u).d(T_u^{m-1},T_u)]$$

$$+\gamma d(u,T_u^{m-1}) + \eta\,[d(u,T_u) + d(T_u^{m-1},u)] + \delta\,[d(u,u) + d(T_u^{m-1},T_u)]$$

$$\le \alpha\left[\frac{\{d\left(u,T_u\right) d^2\,(u,T_u)\}}{d^2\,(u,T_u)}\right] + \beta d(u,T_u) + \gamma\, d(u,T_u^{m-1}) + \eta\,[d(u,T_u) + d(T_u^{m-1},u)]$$

$$+ \delta\, d(T_u^{m-1},T_u)$$

$$d(u,T_u) \le \alpha(u,T_u) + \beta d(u,T_u) + \gamma\, d(u,T_u^{m-1}) + \eta\,[d(u,T_u) + d(T_u^{m-1},u)]$$

$$+\delta\,[d(T_u^{m-1},u) + d(u,T_u)] \qquad (3.2.2)$$

$$d(u,T_u) \le (\alpha + \beta + \eta + \delta)d(u,T_u) + (\gamma + \eta + \delta)d(u,T_u^{m-1}) \qquad (3.2.3)$$

$$d(u,T_u) \le K d(u,T_u^{m-1}) \qquad (3.2.4)$$

Where $K = \frac{(\gamma+\eta+\delta)}{1-(\alpha+\beta+\eta+\delta)} < 1$

$$[1 - (\alpha + \beta + \eta + \delta)]d(u,T_u) \le (\gamma + \eta + \delta)d(u,T_u^{m-1})$$

$$(\alpha + \beta + 2\eta + 2\,\delta + \gamma) \le 1 \qquad (3.2.5)$$

Thus we write d(u, T_u,) ≤ K^md(u, T_u)

Since $K^m < 1$

Therefore d(u, T_u,) ≤ d(u, T_u)

Which is contradiaction.

Hence $T_u = u$,i.e. u is a fixed point of T. But T has not unique fixed point.

Theorem 3.3: Let,T_1 and ,T_2 be two self maps, defined on a complete metric space (X,d) satisfying the condition ;

$$d(T_1\,x, T_2\,y) \le \alpha\left[\frac{d^3(x_{2n}, T_1x_{2n}) + d^3(x_{2n}, T_2x_{2n-1})}{1+d^2(x_{2n}, T_1x_{2n})}\right]$$

$+\beta\left[d\left(x_{2n-1}, T_2x_{2n-1}\right).\, d\left(x_{2n-1}, T_1x_{2n}\right)\right] + \gamma\, d\,(x_{2n}, x_{2n-1})$

$+\eta\,[d((x_{2n}, T_1x_{2n}) + d(x_{2n-1}, T_2x_{2n-1})]$

$+\delta\,[d(x_{2n}, T_2x_{2n-1}) + d(x_{2n-1}, T_1x_{2n})]$

for all x,y,$x \neq y$and for some α,β,γ,η,δ ∈ [0,1) with (α+2β+2γ+2δ+η)<1.then T has

fixed point in X and by Theorem (3.2) $T_1\ and T_2$ are continuous on X.

Then $T_1\ and T_2$ have a common fixed point.

Proof : We have $d(x_{2n+1}, x_{2n}) = d(T_1x_{2n}, T_2x_{2n-1})$

$$d(x_{2n+1}, x_{2n}) \leq \alpha\left[\frac{d^3(x_{2n}, T_1x_{2n}) + d^3(x_{2n}, T_2x_{2n-1})}{1+d^2(x_{2n}, T_1x_{2n})}\right]$$

$+\beta\left[d\left(x_{2n-1}, T_2x_{2n-1}\right).\, d\left(x_{2n-1}, T_1x_{2n}\right)\right] + \gamma\, d\,(x_{2n}, x_{2n-1})$

$+\eta\,[d((x_{2n}, T_1x_{2n}) + d(x_{2n-1}, T_2x_{2n-1})]$

$+\delta\,[d(x_{2n}, T_2x_{2n-1}) + d(x_{2n-1}, T_1x_{2n})]$(3.3.1)

$$\leq \alpha\left[\frac{\{d(x_{2n}, T_1x_{2n}) + d(x_{2n}, T_2x_{2n-1})\}\{d^2(x_{2n}, T_1x_{2n}) + d^2(x_{2n}, T_2x_{2n-1}) + d(x_{2n}, T_1x_{2n})d(x_{2n}, T_2x_{2n-1})\}}{1 + d^2\,(x_{2n}, T_1x_{2n})}\right]$$

$+\beta\left[d\left(x_{2n-1}, x_{2n}\right).\, d\left(x_{2n-1}, x_{2n+1}\right)\right] + \gamma\, d\,(x_{2n}, x_{2n-1}) + \eta\,[d((x_{2n}, x_{2n+1})$

$+d(x_{2n-1}, x_{2n})] + \delta\,[d(x_{2n}, x_{2n}) + d(x_{2n-1}\ x_{2n+1})]$

$\leq \alpha\left[\frac{\{d(x_{2n}, x_{2n+1})d^2(x_{2n}, x_{2n+1})\}}{d^2(x_{2n}, x_{2n+1},)}\right] + \beta\left[d\left(x_{2n}, x_{2n+1}\right)\right] + \gamma\, d\,(x_{2n}, x_{2n-1})$

$+\eta\ [d((x_{2n}, x_{2n+1}) + d(x_{2n-1}, x_{2n})] + \delta\,[d(x_{2n-1}, x_{2n}) + d((x_{2n}, x_{2n+1})]$

$\leq \alpha d(x_{2n}, x_{2n+1}) + \beta d(x_{2n}, x_{2n+1}) + \gamma d(x_{2n}, x_{2n-1}) + \eta[d((x_{2n}, x_{2n+1}) + d(x_{2n-1}, x_{2n})]$

$+\delta\,[d(x_{2n-1}, x_{2n}) + d((x_{2n}, x_{2n+1})]$ (3.3.2)

$d(x_{2n+1}, x_{2n}) \leq (\alpha + \beta + \eta + \delta)d(x_{2n}, x_{2n+1}) + (\gamma + \eta + \delta)d(x_{2n-1}, x_{2n})$ (3.3.3)

$$d(x_{2n+1}, x_{2n}) \leq \frac{(\gamma + \eta + \delta)}{1-(\alpha + \beta + \eta + \delta)}\, d(x_{2n-1}, x_{2n})$$

$$d(x_{2n+1}, x_{2n}) \leq K^{2n} d((x_0, x_1)$$

Where $K = \frac{(\gamma + \eta + \delta)}{1-(\alpha+\beta+\eta+\delta)} < 1$

$$[1 - (\alpha + \beta + \eta + \delta)]d(x_{2n+1}, x_{2n}) \leq (\gamma + \eta + \delta)d((x_{2n-1}, x_{2n})$$

$$(\alpha + \beta + 2\eta + 2\,\delta + \gamma) < 1$$

Similarly we can show that

$$(x_{2n+1}, x_{2n+2}) \leq K^{2n+1} d((x_0, x_1)$$

Now it can be easily seen that $\{x_n\}$ is a Cauchy sequence. Let $x_n \to u$ then the subsequence$\{x_{n_p}\}$ also converges to u, for $n_p = 2p$.

Now $T_1T_2(u) = T_1T_2 \lim_{p\to\infty} x_{n_p}$

$= \lim_{p\to\infty} x_{n_p} + 1$

= u

We now show that $T_2(u) \neq u$

If $T_2(u) \neq u$,then

d (u,T_2u) =d(T_1T_2u , T_2u)

d(u,T_2u)$\leq \alpha \left[\frac{d^3\,(T_2u\,,T_1T_2u)+d^3\,(T_2u\,\,,T_2u)}{1+d^2\,(T_2u\,\,,\,\,T_1T_2u)}\right]$ + β$[d(u\,,\,T_2u).\,d(u\,,T_1T_2u)]$ + γ d $(T_2u\,,u\,\,)$

+η $[d((T_2u\,,T_1T_2u) + d(u\,,\,T_2u)]$+ δ $[d(T_2u\,,T_2u) + d(u,T_1T_2u)$ (3.3.3)

$$\leq \alpha \left[\frac{d(T_2u\,,u) + d(T_2u\,\,,T_2u)\{(d^2T_2u\,,u) + d^2(T_2u\,\,,T_2u) + d(T_2u\,,u).\,d(T_2u\,\,,T_2u)\}}{1 + d^2\,(T_2u\,,u)}\right]$$

+β$[d(u\,,T_2u).\,d(u\,,u)]$ +γ d$(T_2u\,,u)$ + η $[d((T_2u\,,u) + d(u\,,\,T_2u)]$+ δ $d(T_2u\,,T_2u\,)$ +

$d(u,u)]$

$\leq \alpha \left[\frac{d(T_2u\,,u)\{d^2\,(T_2u\,,u)\}}{d^2\,(T_2u\,\,,\,\,u)}\right]$ +γd (T_2u,u) + η $[d((T_2u\,,u) + d(u\,,\,T_2u)] \leq \alpha d(T_2u\,,u)$+γ d

$(T_2u\,\,,u)$ + η $[d((T_2u\,,u) + d(u\,,\,T_2u)]$□

d(u,T_2u) ≤$(\alpha + \gamma + 2\eta)$ d(T_2u,u) (3.3.4)

which is contradiction.

∴ $(\alpha + \beta + 2\eta + 2\,\delta + \gamma) < 1$,so $(\alpha + \gamma + 2\eta) \leq 1$.

Hence we have $T_2u = u$

Now $T_1T_2u = T_2u = u$

Thus u is the common fixed point of $T_1 and T_2$,But uniqueness is not possible.

Conclusion: In this paper we proof some theorems of fixed points in complete metric space.

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