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Structural Analysis of Epicyclic Gear Trains

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Abstract* - Epicyclic gear trains belong to a special class of geared kinematic chains. Enumeration of epicyclic gear trains (EGT's) is a complex process. A large number of graphs can be formed from a set of given elements. Constraints are required to be imposed on the elements to develop feasible alternatives representing EGT's. The enumeration process invariably involves detection of isomorphism among several geared kinematic chains generated. Several methods are presented by researchers for detection of isomorphism and other structural properties of kinematic chains (K-chains). Methods for detection of isomorphism among geared k-chains are few and have been examined with limited number of cases. Thorough analysis of existing methods is limited in literature. Graph theory has played important role in the study of mechanisms. Several algorithms of graph theory find their application in deriving various properties of geared kinematic chains. These algorithms are explored and used. An algorithm is developed and implemented on computer for automation of the structural analysis process. With the help of the computer based algorithm a total of 108 geared K-chains are were analyzed and results are presented. These geared chains are further analysed for identification of equivalents elements for possible alternatives designs.

Keywords: Geared Kinematic chains, mechanisms, inversion, and genetic algorithm.

I. INTRODUCTION

Design is fundamental to every branch of engineering. Study of engineering design is concerned with creation of something new or arranges existing things in a new way to satisfy a recognized need of the society. Mechanical engineering design deals with synthesis of various parts and mechanisms required for generation of motion for a given situation. Therefore, synthesis and analysis of mechanisms and machines becomes an inherent part in mechanical engineering design course. Over a period of time with advancement in technology present machine design process has changed a lot and requires greater skill set to design complex parts and machineries which are now part of the modern society. This has led to the origin of this project. For a better understanding of complex machines newer tools such graph theory and genetic algorithm are used and with the help of these tools analyzing Mechanisms and machines become simple. Present work deals with the application of graph theory and genetic algorithm in machine design. The need for compact gear boxes laid to the development of planetary gear trains. Planetary gear trains are compact and used to obtain high velocity ratios.

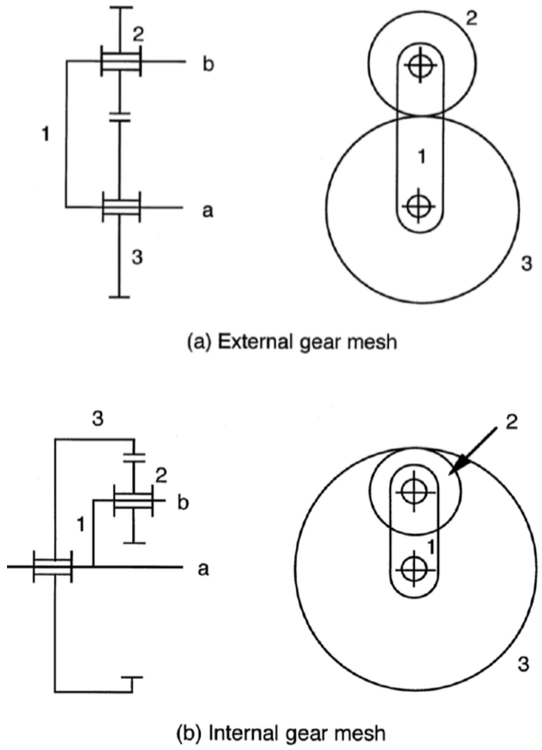


Fig. 1. Meshing of Gear Trains

Structure and analysis of planetary gear trains is first studied by Levai [2] in the year 1968. Synthesis of kinematic structure of geared kinematic chains and other mechanisms was proposed by Buchsbaum and Freudenstein [3].

Smith [4] analyzed epicyclic gear trains via the vector loop approach. Tsai [5] proposed a method for the application of the linkage characteristic polynomial to the topological synthesis of epicyclic gear train. The creation of non-fractionated two-degree of- freedom epicyclic gear trains was given by Tsai and Lin [6].

Hsu [7] gaves a method for the application of generalized kinematic chains to the structural synthesis of non-fractionated epicyclic gear trains.

Dr Rao [8] presented a paper on generation of epicyclic gear trains. Dr Rao emphasized that the works reported so far deals with the generation of distinct graphs and then go for reverse transformation. It is desirable to know the characteristics inherent to the structure so that best chain can be selected from the numerous distinct chains (gear trains) with the same number of links and degree-of-freedom.

II. PROPOSED METHODOLOGY

Structural properties of kinematic chains of EGT's make it easier for the designer to select a particular EGT's for a specified task. Studies in structural analysis were largely concentrated on finding distinct gear trains of a given geared kinematic chain. Rao and his co-workers reported several ideas related to structure- based performance assessment of mechanisms.

The algorithm for structural analysis consists of the following steps.

1. Provide the adjacency matrices of geared kinematic chains. As an example

Consider 4-element geared kinematic chain shown in Fig.2.

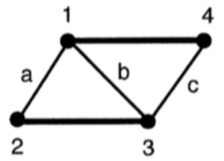


Fig. 2. Element Epicyclic Gear Train Graph

One-adjacency matrix A_1 of the chain will be

$$A_1 = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

Half-adjacency matrix $A_{1/2}$ will be

$$A_{1/2} = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

One-adjacency rotation matrix A_R will be

$$A_R = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

2. Computation of fitness matrices. An algorithm was written to constraints fitness matrices from the given adjacency matrix. Fitness matrices for the given Fig.2 as calculated by the algorithm are

$$F_1 = \begin{bmatrix} 0 & 3 & 4 & 1 \\ 3 & 0 & 1 & 2 \\ 4 & 1 & 0 & 3 \\ 1 & 2 & 3 & 0 \end{bmatrix}$$

$$F_{1/2} = \begin{bmatrix} 0 & 2 & 2 & 2 \\ 2 & 0 & 2 & 2 \\ 2 & 2 & 0 & 2 \\ 2 & 2 & 2 & 0 \end{bmatrix}$$

$$F_R = \begin{bmatrix} 0 & 3 & 4 & 1 \\ 3 & 0 & 1 & 2 \\ 4 & 1 & 0 & 3 \\ 1 & 2 & 3 & 0 \end{bmatrix}$$

3. Formation of element strings. Once the fitness matrices are computed it is easy to obtain the element fitness string. Element fitness string is defined here as concatenation of the equivalent fitness strings along with the fitness sum. Consider, for example, geared K-chain of Fig.2. Concatenated element fitness string for F_1 is

([6 -3210] [6 -3210] [8 -4310] [8 -4310])

and concatenated element fitness string for $F_{1/2}$ is

([6 -2220] [6 -2220] [6 -2220] [6 -2220])

Similarly concatenated element fitness string for F_R is

([6 -3210] [6 -3210] [8 -4310] [8 -4310])

4. Formation of chain strings. Chain string is defined as the concatenation of the unique element strings for all fitness matrices. Therefore, chain string for the chain shown in Fig.2. is

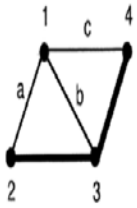
F1 (2[6 - 3210] 2[8 - 4310])

F1/2 (4[6 - 2220])

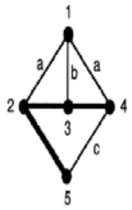
FR (2[6 - 3210] 2[8 - 4310])

III. DETECTION OF DISTINCT MECHANISMS

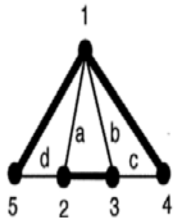
Genetic algorithm was found to be a useful tool in the structural analysis of geared kinematic chains. The algorithms were applied to different sets of geared kinematic chains. A total of 108 chains collected from literature sources were chosen for analysis of A_1, $A_{1/2}$ and A_R adjacency matrices for each geared K-chain were formed. An algorithm written in MATLAB was used for structural analysis of these chains. Results from program were checked manually to identify errors and to validate the results. Out of 108 chains 43 chains have been found to have distinct fitness string. The duplicacy of fitness strings was obvious as only vertex properties are taken in account for calculation of fitness string. In order to uniquely identify edges the methods needs to be explored. Present work provides the result of 108 chains as derived by the method. The method has successfully able to distinguish displacement and rotational isomorphism among geared K-chain.



$$\begin{pmatrix} F_1\ [12-4440]3[4-4000] \\ F_{1/2}\ [4-2110]2[4-3100][8-3320] \\ F_R\ [12-4440]3[4-4000] \end{pmatrix}$$



$$\begin{pmatrix} F_1\ [10-53110][15-54420]2[7-42100][9-32220] \\ F_{1/2}\ 2[10-43210][6-22110][7-32110] \\ F_R\ [20-55550]4[5-50000] \end{pmatrix}$$



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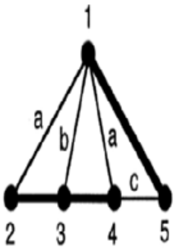

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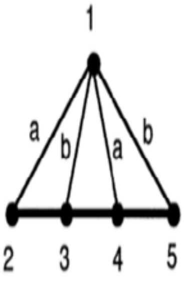

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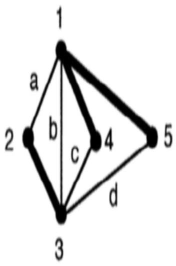
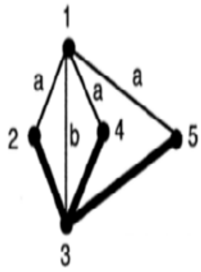
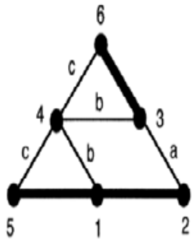


$$\begin{pmatrix} F_1\ [10-53110][15-54420]2[7-42100][9-32220] \\ F_{1/2}\ [12-33330]2[7-32200]2[9-32220] \\ F_R\ [10-53110][15-54420]2[7-42100][9-32220] \end{pmatrix}$$



$$\begin{pmatrix} F_1\ [20-55550]4[5-50000] \\ F_{1/2}\ [15\quad 44430]3[5\quad 41000][6\quad 31110] \\ F_R\ [20-55550]4[5-50000] \end{pmatrix}$$



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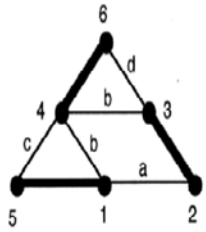

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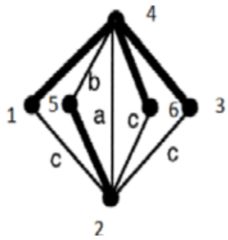

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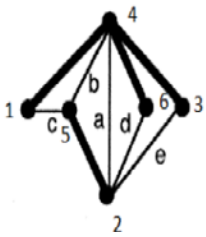

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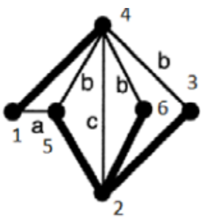

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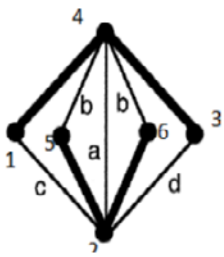

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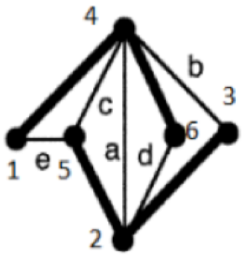

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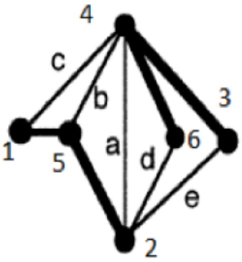

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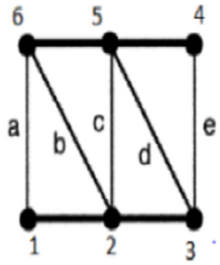

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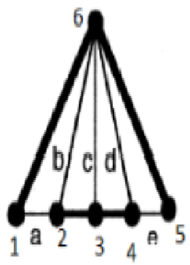

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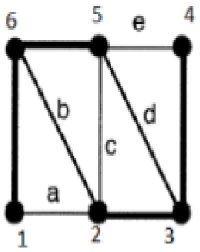

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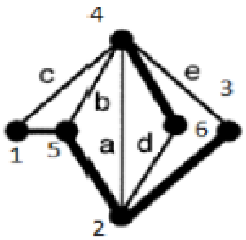

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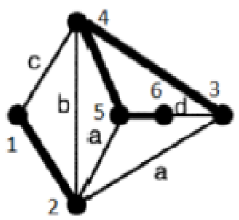

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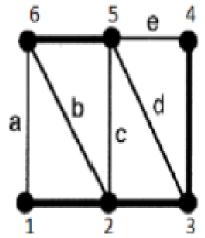

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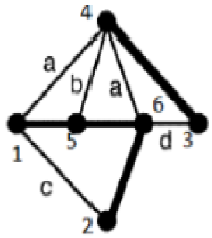

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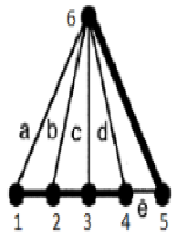

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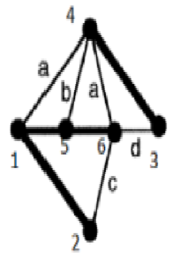

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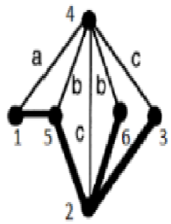

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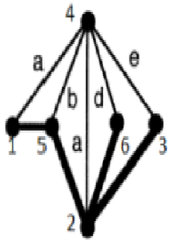

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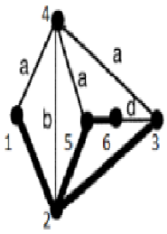

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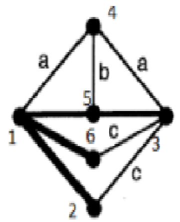

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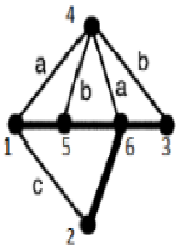

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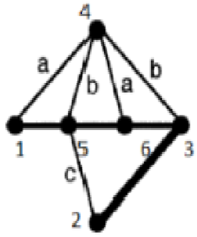
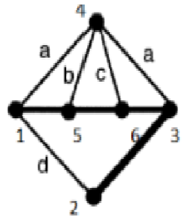
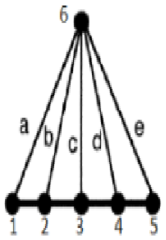


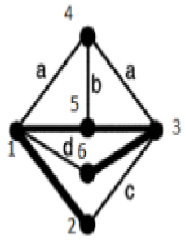
$$\begin{pmatrix} F_1\ [12-332220][12-631110] \\ [24-655530]3[8-521000] \\ F_{1/2}\ 2[10-332110][12-442110] \\ 2[14-433220][8-222110] \\ F_R\ [30-666660]5[6-600000] \end{pmatrix}$$



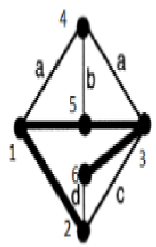
$$\begin{pmatrix} F_1\ [12-332220][12-631110] \\ [24-655530]3[8-521000] \\ F_{1/2}\ 2[10-332110][12-442110] \\ 2[14-433220][8-222110] \\ F_R\ [10-431110]2[10-522100] \\ [12-432210][16-543220][22-555430] \end{pmatrix}$$



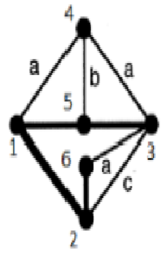
$$\begin{pmatrix} F_1\ [30-666660]5[6-600000] \\ F_{1/2}\ 2[10-332110][12-442110] \\ 2[14-433220][8-222110] \\ F_R\ [30-666660]5[6-600000] \end{pmatrix}$$



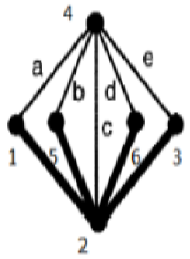
$$\begin{pmatrix} F_1\ [10-422110]2[12-332220] \\ 2[14-533210][18-554220] \\ F_{1/2}\ 2[10-332110][12-442110] \\ 2[14-433220][8-222110] \\ F_R\ 3[12-422220]3[16-444220] \end{pmatrix}$$



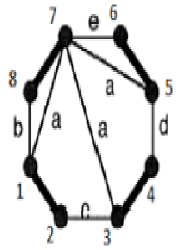
$$\begin{pmatrix} F_1\ 2[10-432100][12-432210] \\ [12-541110][16-433330][20-544430] \\ F_{1/2}\ 2[10-332110][12-442110] \\ 2[14-433220][8-222110] \\ F_R\ [12-333210][12-432210] \\ [14-433220][16-533320] \\ [16-543220][18-444330] \end{pmatrix}$$



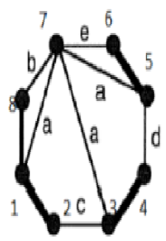
$$\begin{pmatrix} F_1\ 4[10-422200]2[18-644220] \\ F_{1/2}\ [15-433320][7-221110] \\ 2[7-321100][9-322110][9-421110] \\ F_R\ [10-431110]2[10-522100] \\ [12-432210][16-543220][22-555430] \end{pmatrix}$$



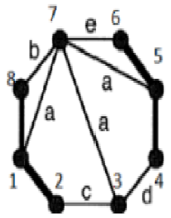
$$\begin{pmatrix} F_1\ [30-666660]5[6-600000] \\ F_{1/2}\ [24-555540] \\ 4[6-510000][8-411110] \\ F_R\ [30-666660]5[6-600000] \end{pmatrix}$$



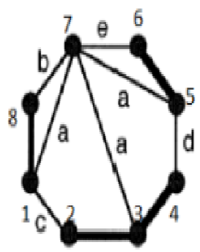
$$\begin{pmatrix} F_1\ [14-52221110] \\ 3[18-33332220] \\ 3[20-63332210][32-66653330] \\ F_{1/2}\ 8[14-22222220] \\ F_R\ [16-43222210] \\ 2[16-43322200][18-44322210] \\ [20-54333110][24-54333330] \\ [26-65443220][28-65444320] \end{pmatrix}$$



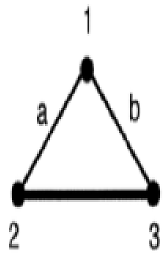
$$\begin{pmatrix} F_1\ 3[12-62211000] \\ 2[18-43322220] \\ 2[18-73321110][40-77666440] \\ F_{1/2}\ 2[12-32222100]4[14-32222210][20-33333320] \\ [8-21111110] \\ F_R\ 3[14-53321000] \\ [18-53322210][18-64321110] \\ [22-44433310][24-64333320] \\ [36-66555540] \end{pmatrix}$$



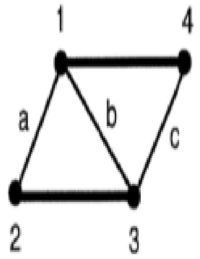
$$\begin{pmatrix} F_1\ 4[12-62220000] \\ 2[16-44222200][24-84422220] \\ [40-86666440] \\ F_{1/2}\ 4[12-33221100] \\ 2[20-43333220]2[8-22111100] \\ F_R\ 4[16-55221100]2[16-66111100]2[36-66555540] \end{pmatrix}$$



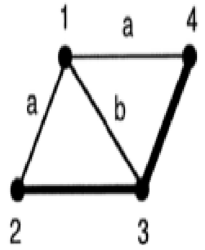
$$\begin{pmatrix} F_1\ 3[12-62211000] \\ 2[18-43322220] \\ 2[18-73321110][40-77666440] \\ F_{1/2}\ 2[12-32222100] \\ 4[14-32222210] \\ [20-33333320][8-21111110] \\ F_R\ 3[16-54322000] \\ [18-54222210] \\ [20-43333310][22-74322220] \\ [28-54444430][36-75555540] \end{pmatrix}$$



$$\begin{pmatrix} F_1\ 2[3-300][6-330] \\ F_{1/2}\ [2-110]2[3-210] \\ F_R\ 2[3-30][6-330] \end{pmatrix}$$



$$\begin{pmatrix} F_1\ 2[6-3210]2[8-4310] \\ F_{1/2}\ 4[6-2220] \\ F_R\ 2[6-3210]2[8-4310] \end{pmatrix}$$



$$\begin{pmatrix} F_1\ [12-4440]3[4-4000] \\ F_{1/2}\ [4-2110]2[4-3100] \\ [8-3320] \\ F_R\ [12-4440]3[4-4000] \end{pmatrix}$$

IV. CONCLUSION

An algorithm was developed using the concepts of graph theory and genetic algorithm for Structural analysis of EGT's. With the help of the algorithm complete set of 108 geared K-chains were analyzed. Comparison of fitness strings of two geared kinematic chains reveals isomorphism if any between the geared kinematic chains. The proposed algorithm has successfully detected displacement and rotational isomorphism among geared K-chains.#

REFERENCES

[1] Norton, R. L. Kinematics and Dynamics of Machinery. Tata McGraw Hill Education Private Limited, New Delhi, 2009.

[2] Levai, Z Structure and Analysis of Planetary Gear Trains . Journal of Mechanisms 3: 131-148, 1968.

[3] Buchsbaum, F; Freudenstein, F. Synthesis of Kinematic Structure of Geared Kinematic Chains and other Mechanisms. Journal of Mechanisms, 5: 357-392,1970.

[4] Smith, D. of Epicyclic Gear Trains via the Vector Loop Approach. Sixth Applied Mechanisms Conference, Denver, 1979.

[5] Tsai, L.W. An Application of the Linkage Characteristic Polynomial to the Topological Synthesis of Epicyclic Gear Trains . ASME Journal of Mechanisms,Transmissions and Automation in Design, 109: 329-336, 1987.

[6] Tsai, L.W; Lin, C.C. The Creation of Non-fractionated Two-Degree of- Freedom Epicyclic Gear Trains. ASME Journal of Mechanisms, Transmissions, and Automation in Design, 111: 524-529, 1989.

[7] Hsu, C.H. Application of Generalized Kinematic Chains to the Structural Synthesis of Non-Fractionated Epicyclic Gear Trains, in Proceedings of the 22nd ASME Mechanism. 22nd ASME Mechanisms Conference, 46: 451? 458, Scottsdale dale, 1992.

[8] Rao, A. C. A Genetic Algorithm for Epicyclic Gear Trains. Mechanism and Machine Theory, 38:135- 147, AP, 2003.