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# Common Fixed Point Theorem in Intuitionistic Fuzzy Metric Space by Employing Common E.A. Property

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***Abstract:The goal of this manuscript is to make use of the property (E.A.) and the common property (E.A.) to prove some common fixed point results for integral type mapping in intuitionistic fuzzy metric space. our results generalize and extend several relevant common fixed point theorems from the literature.***

***Keywords:Common E.A. property, t-norm, t-conorm, Intuitionistic fuzzy metric space.***

***Mathematics Subject Classification:Primary 47H10, 54H25.***

## I. INTRODUCTION AND PRELIMINARIES

In 1906 Frechet [7] introduced the concept of metric space which is defined as " A nonempty abstract set $X$associated with a function of two variables allow us to calculate the distance between any two points" Zadeh was instrumental with a new idea of fuzzy sets in 1965 [27].Thereafter, it was developed extensively by many authors, which also includes interesting applications of this theory in diverse areas. To use this concept in topology and analysis several researchers have defined fuzzy metric spaces in several ways.Kramosil and Michalek [16] establish the abstract notion of fuzzy metric space that can be contemplate as generalization of the statistical metric space.Afterwards George and Veeramani [9] prove some known results by defining the Hausdroff topology and faintly altered the fuzzy metric concept.Atanassov [3] defined the Intuitionistic fuzzy set generalized form of fuzzy set.The strength of fuzzy mathematics lies in its noted and fruitful applications especially outside mathematics. Even the various concepts of fuzzy topology have already found vital applications in quantum particle physics particularly in connection with both string and $\varepsilon^{\infty}$theory which were studied and formulated by EI Naschie [8].Most recently, Gregori et al. [11] have furnished several interesting examples of fuzzy metrics in the sense of George and Veeramani [9] and have also utilized such fuzzy metrics to color image processing. In 1997,Çoker proposed the abstraction of intuitionistic fuzzy topological space [6]. Later on the concept of $t-$norm and $t-$conorm initiated by Schweizer and Sklar [23] which was very constructive in introducing the intuitionistic fuzzy metric space.

The origin of fixed point theory in fuzzy metric space is often traced back to the paper of Grabiec [10] wherein the extended classical fixed point theorem of Banach and Edelstein to complete and compact fuzzy metric spaces respectively. Subrahmanyam [24]gave an extension of Jungck's [14] common fixed point theorem to fuzzy metric spaces. In this continuation Vasuki [26] proved fuzzy analogue of a result due to pant[20] in complete fuzzy metric space which has been further improved by Imdad and Ali [12] employing the notions of R-weak commutativity of type $(A_f), (A_g)$ and (P). [12],[13],[18],[19],[25].

Branciari [5] initiated a study of contractive conditions of integral type, giving an integral version of the Banach Contraction Principle , that could be extended to more general contractive conditions. To be precise, Branciari established the following theorem.

Let $(X,d)$ be a complete metric space, $c \in\ ]0,1[$ and let $f: X \to X$ be a mapping such that for each $x, y \in X$,

$$\int_0^{d(fx,fy)} \zeta(s)ds \le c\int_0^{d(x,y)} \zeta(s)ds$$

where $\zeta: [0,+\infty[\ \to [0,+\infty[$ is a lebesgue integral mapping which is summable on each compact subset of $[0,+\infty[$ and such that $\varepsilon > 0$.

$$\int_0^{\varepsilon} \zeta(s)ds > 0$$

Then, $f$ admits a unique fixed point $a \in X$ such thatfor each $x \in X, f^n x \to a$ as $n \to \infty$.

Bhardwaj [4] proved some fixed point theorem for contractive condition of integral type in complete metric space.Further discussion is made in [28-32] forthis mappings.

**Definition 1.1**[23]"A binary operation $*$: $[0,1] \times [0,1] \to [0,1]$ is called a continuous $t$-norm if$*$ hold the following conditions:

- $a * b = b * a$,
- $a * (b * c) = (a * b) * c$
- $*$ is continuous,
- $a * 1 = a \ \forall a \in [0,1]$,
- $a * b \leq c * d$where $a \leq c$ and $b \leq d$ for all $a, b, c, d \in [0,1]$."

**Definition 1.2**[23]"A $t$-conorm $\diamond$ is a binary operation on $[0,1]$ which hold the following conditions:

- $a \diamond b = b \diamond a$,
- $a \diamond (b \diamond c) = (a \diamond b) \diamond c$,
- $\diamond$ is continuous,
- $a \diamond 0 = a \ \forall \ a \in [0,1]$,
- $a \diamond b \leq c \diamond d$where$a \leq c$ and $b \leq d$ for each $a, b, c, d \in [0,1]$."

Using $t$-norm and $t$-conorm Park [21] in 2004conceptualize theintuitionistic fuzzy metric space, which is defined as follows:

**Definition 1.3**[21]A $5-$tuple $(X, M, N, *, \diamond)$ is intuitionistic fuzzy metric space. If $X$ is an random set, $*$ a continuous $t$-norm and $\diamond$ a continuous $t$-conorm and $M, N$ are fuzzy sets on $X^2 \times [0, \infty)$fulfill the following conditions $\forall x, y, z \in X \ s, t > 0$

- $M(x, y, t) + N(x, y, t) \leq 1$,
- $M(x, y, t) > 0$,
- $M(x, y, t) = 1 \ \forall \ t > 0 \ iff \ x = y$,
- $M(x, y, t) = M(y, x, t)$,
- $M(x, z, t) * M(y, z, s) \leq M(x, y, t + s)$,
- $M(x, z, \cdot) : (0, +\infty) \to [0, 1]$,
- $lim_{t \to \infty} M(x, y, t) = 1$,
- $N(x, y, t) < 1$,
- $N(x, y, t) = 0, iff \ x = y$,
- $N(x, y, t) = N(y, x, t)$,
- $N(x, z, t) \diamond N(y, z, s) \geq N(x, y, t + s)$,
- $N(x, z, \cdot): (0, +\infty) \to [0,1]$,
- $lim_{t \to \infty} N(x, y, t) = 0$.

Then $(M, N)$ is known an intuitionistic fuzzy metric on (IFMS) $X$. The function $M(x, y, t)$ and $N(x, y, t)$indicate the nearness degree and the non-nearness degree between $x$ and $y$in respect to $t$, respectively.

**Example:1.1** Let $\psi: X \to R^+$ be a one-one function and let $\xi: R^+ \to [0,\infty)$ be an increasing continuous function. For fixed $\gamma, \delta > 0$defined intuitionistic fuzzy as fuzzy as

$$\mathbb{M}(x,y,t) = \left(\frac{(min\{\psi(x),\psi(y)\})^\gamma + \xi(t)}{(max\{f(x),f(y)\})^\gamma + \xi(t)}\right)^\delta$$

$$\mathbb{N}(x,y,t) = 1 - \mathbb{M}(x,y,t) = 1 - \left(\frac{(min\{\psi(x),\psi(y)\})^\gamma + \xi(t)}{(max\{f(x),f(y)\})^\gamma + \xi(t)}\right)^\delta$$

Then $(X, \mathbb{M}, \mathbb{N}, *, \diamond)$ is said to be IFMS on $X$.

After that many authors studied this concept and generated fixed point outcomes in intuitionistic fuzzy metric space. In 2006 Saadati and park [22] brought up the Intuitionistic fuzzy topological spaces.

**Definition 1.4** A sequence $\{x_n\}$ is said to be convergent to $x \in X$ in IFMS if$\forall t > 0$there is some $n_p \in$ N such that $\int_0^{\mathbb{M}(x_n,x,t)=1} \zeta(t)dt = 1$and $\int_0^{\mathbb{N}(x_n,x,t)=0} \zeta(t)dt = 0\ \forall n \geq n_p$ if $n \to \infty$

**Definition1.5** If $(X, \mathbb{M}, \mathbb{N}, *, \diamond)$ if IFMS and $\{x_n\}\{y_n\}$ are sequences in $X$ such that $x_n \to x$, $y_n \to y$then $\mathbb{M}(x_n, y_n, t) \to \mathbb{M}(x,y,t)$ and $\mathbb{N}(x_n, y_n, t) \to \mathbb{N}(x,y,t)$ for every continuity point $t$ of $\mathbb{M}(x,y,.)$ and $\mathbb{N}(x,y,.)$.

**Definition 1.6**[15] A pair $(\psi, \rho)$ of self mapping of a IFMS is said to be compatible if for all $t > 0$,

$\int_0^{\mathbb{M}(\psi\rho x_n, \rho\psi x_n, t)=1} \zeta(t)dt = 1, \int_0^{\mathbb{N}(\psi\rho x_n, \rho\psi x_n, t)=1} \zeta(t)dt = 0,$

where$\{x_n\}$ is a sequence in $X$ such that $\lim_{n\to\infty} \psi x_n = lim_{n\to\infty} \rho x_n = z$ for some $z \in X$.

**Definition 1.7**A pair $(\psi, \rho)$ of self mapping of IFMS$(X, \mathbb{M}, \mathbb{N}, *, \diamond)$ is said to be non-compatible if there exists at least one sequence $\{x_n\}$ in $X$such that $lim_{n\to\infty} \psi x_n = lim_{n\to\infty} \rho x_n = z$ for some $z \in X$ but $\int_0^{\mathbb{M}(\psi\rho x_n, \rho\psi x_n, t)=1} \zeta(t)dt \neq 1$, $\int_0^{\mathbb{N}(\psi\rho x_n, \rho\psi x_n, t)=1} \zeta(t)dt \neq 0$ or non-existent for at least one $t > 0$ as $n$approaches to infinity.

Motivated from [1] one can have the following (as utilized in [18])

**Definition 1.8** A pair $(\psi, \rho)$ of self mapping of IFMS satisfy the property (E.A.) if there exists a sequence $\{x_n\}$ in $X$ such that for all $t > 0$

$$\int_0^{\mathbb{M}(\psi x_n, \rho x_n, t)} \zeta(t)dt = 1$$

$$\int_0^{\mathbb{N}(\psi x_n, \rho x_n, t)} \zeta(t)dt = 0$$

clearly, a pair of nontrivial compatible mappings (as well as non-compatible mapping) enjoys the property (E.A.)

Also the lines of Liu et al. [17] one can have the following.

**Definition 1.9** Two pairs $(\psi, \rho)$ and $(\mu, \xi)$ of self mappings of IFMS are said to satisfy the common property (E.A.) if there exists two sequences $\{x_n\}$ and $\{y_n\}$ in $X$ such that for all $t > 0$

$$\int_0^{\mathbb{M}(\psi x_n, \rho x_n, t)} \zeta(t)dt = \int_0^{\mathbb{M}(\mu x_n, \xi x_n, t)} \zeta(t)dt = 1$$

$$\int_0^{\mathbb{N}(\psi x_n, \rho x_n, t)} \zeta(t)dt = \int_0^{\mathbb{N}(\mu x_n, \xi x_n, t)} \zeta(t)dt = 0$$

**Definition 1.10** A pair $(\psi, \rho)$ of self mapping of a non-empty set $X$ is said to be weakly compatible if $\psi x = \rho x$ for $x \in X$implies $\psi\rho x = \rho\psi x$

**Definition 1.11**[2] Two finite families of self mapping $\{A_i\}_{i=1}^m$ and $\{B_i\}_{i=1}^m$ of a nonempty set $X$ are said to be pairwise commuting if

$$A_i A_j = A_j A_i i, j \in \{1,2,\dots,m\}$$

$$B_k B_l = B_l B_k k, l \in \{1,2,\dots,n\}$$

$A_i B_k = B_k A_i i \in \{1,2,\dots,m\}$ and $k \in \{1,2,\dots,n\}$

The purpose of this paper is to emphasize the role of the property (E.A.) and the common property (E.A.) in the existence of a common fixed point for contractive mappings in fuzzy metric spaces. Our results generalize and extend several results from the existing literature

Our results involve the class $\Phi$ of all mapping $\phi$: $[0,1] \to [0,1]$ and $\Psi$: $[0,1] \to [0,1]$satisfy the following properties:

(a) $\phi$, $\Psi$is continuous and nondecreasingand nonincreasing respectively on $[0,1]$

(b) $\phi(x) > x\forall\, x \in (0,1)$ and $\Psi(x) < x$

we note that if $\phi, \Psi \in \Phi$, then $\phi(1) = 1, \Psi(1) = 1$and $\phi(x) \geq x$, $\Psi\,(x) \leq x\ \forall x \in [0,1]$,

## II. MAIN RESULT

**Lemma 2.1** Let $\psi, \mu$, ρ and $\xi$ be four self mappings of a IFMS $(X, \mathbb{M}, \mathbb{N}, *, \diamond)$ satisfying the following conditions:

(1) the pair $(\psi, \rho)$ or $(\mu, \xi)$ satisfy the property (E.A.)

(2) $\psi(x) \subset \xi(x)$ or $\mu(x) \subset \rho(x)$

(3) $\mu(y_n)$converges for every sequence $\{y_n\}$ in $X$whenever $\xi(y_n)$converges (or $\psi(y_n)$converges for every sequence $\{y_n\}$ in $X$whenever $\rho(y_n)$converges.

(4) $\forall x, y \in X, x \neq y \exists t > 0 : 0 < \mathbb{M}(x, y, t) < 1$, for some $\phi \in \Phi$

,

$$\int_0^{\mathbb{M}(\psi x, \mu y, t)} \zeta(s)ds \geq \int_0^{\phi(min\{\mathbb{M}(\rho x, \xi y, t), \mathbb{M}(\psi x, \rho x, t), \mathbb{M}(\mu y, \xi y, t), \mathbb{M}(\psi x, \xi y, t), \mathbb{M}(\rho x, \mu y, t)\})} \zeta(s)ds$$

and$\int_0^{\mathbb{N}(\psi x, \mu y, t)} \zeta(s)ds \leq \int_0^{\Psi(max\{\mathbb{N}(\rho x, \xi y, t), \mathbb{N}(\psi x, \rho x, t), \mathbb{N}(\mu y, \xi y, t), \mathbb{N}(\psi x, \xi y, t), \mathbb{N}(\rho x, \mu y, t)\})} \zeta(t)dt(1)$

then the pairs $(\psi, \rho)$ and $(\mu, \xi)$share the common property (E.A)

Proof: Since the $(\psi, \rho)$enjoys the property E.A. there exists a sequence $\{x_n\}$ in $X$ such that

$\lim_{n\to\infty} \psi x_n = lim_{n\to\infty} \rho x_n = z$ for some $z \in X$

such that $\int_0^{\mathbb{M}(\psi x_n, \rho x_n, t)} \zeta(t)dt = 1$ , $\int_0^{\mathbb{N}(\psi x_n, \rho x_n, t)} \zeta(t)dt = 0$Since $\psi(x) \subset \xi(x)$ for each $\{x_n\}$there exists $\{y_n\}$ in $X$ such that $\psi x_n = \xi y_n$ and hence $\lim_{n\to\infty} \psi x_n = \lim_{n\to\infty} \xi x_n = z$. Thus in all, we have $\psi x_n \to z$, $\rho x_n \to z$ and $\xi x_n \to z$. Now,we assert that $\int_0^{\mathbb{M}(\mu x_n, z, t)} \zeta(t)dt = 1 \int_0^{\mathbb{N}(\mu x_n, z, t)} \zeta(t)dt = 0$ as $n \to \infty$

Suppose $lim_{n\to\infty} \rho x_n = p \neq z$

$$\int_0^{\mathbb{M}(\psi x_n, \mu x_n, t)} \zeta(t)dt \geq \int_0^{\phi(min\{\mathbb{M}(\rho x_n, \xi y_n, t), \mathbb{M}(\psi x_n, \rho x_n, t), \mathbb{M}(\mu y_n, \xi y_n, t), \mathbb{M}(\psi x_n, \xi y_n, t), \mathbb{M}(\rho x_n, \mu y_n, t)\})} \zeta(t)dt$$

$$\int_0^{\mathbb{N}(\psi x, \mu y, t)} \zeta(t)dt \leq \int_0^{\Psi(max\{\mathbb{N}(\rho x_n, \xi y_n, t), \mathbb{N}(\psi x_n, \rho x_n, t), \mathbb{N}(\mu y_n, \xi y_n, t), \mathbb{N}(\psi x_n, \xi y_n, t), \mathbb{N}(\rho x_n, \mu y_n, t)\})} \zeta(t)dt$$

for all $n \in$ N as $n \to \infty$

then

$$\int_0^{\mathbb{M}(z,p,t)} \zeta(t)dt \geq \int_0^{\phi(\mathbb{M}(z,l,t))} \zeta(t)dt$$

$$\int_0^{\mathbb{N}(z,p,t)} \zeta(t)dt \leq \int_0^{\phi(\mathbb{N}(z,l,t))} \zeta(t)dt$$

As $z \neq p$, we have $0 < \mathbb{M}(z, p, t_0) < 1$ for some $t_0 > 0$. Since $\mathbb{M}(z, p, .)$ is left continuous and is also nondecreasing, it can have at the most countably many points of discontinuity. Let on contrary that $t_0$ is a point of continuity of

$\mathbb{M}(z,p,.)$Then in view of condition $\phi(\mathbb{M}(z,p,t_0)) > M(z,p,t_0)$, and $\Psi(\mathbb{N}(z,p,t_0)) < \mathbb{N}(z,p,t_0)$ which is contradiction, implying thereby $z = p$. This shows that the pair $(\psi, \rho)$ and $(\mu, \xi)$share the common property (E.A.)

**Theorem: 2.1** Let $\psi, \mu, \rho$ and $\xi$ be four self mapping of IFMS $(X, \mathbb{M}, \mathbb{N}, *, \diamond)$ satisfying the equation (1). Suppose that

The pairs $(\psi, \rho)$ and $(\mu, \xi)$share the common property (E.A.).

$\rho(X)$and $\xi(X)$ are closed subset of $X$.

Then the pair $(\psi, \rho)$ and $(\mu, \xi)$have a coincidence point. Moreover $\psi, \mu, \rho$ and $\xi$have a unique common fixed point in $X$provided both the pairs $(\psi, \rho)$ and $(\mu, \xi)$ are weakly compatible.

Proof: Since the pairs $(\psi, \rho)$ and $(\mu, \xi)$share the common property (E.A.) therefore the exists two sequences $\{x_n\}$ and $\{y_n\}$ in $X$ such that

$\lim_{n\to\infty} \psi x_n = \lim_{n\to\infty} \rho x_n = \lim_{n\to\infty} \mu x_n = \lim_{n\to\infty} \xi x_n = z$ for some $z \in X$

Since $\rho(X)$ is a closed subset of $X$. therefore $\lim_{n\to\infty} \rho\, x_n = z \in \rho(X)$. Hence there exists a point $v \in X$ such that $\rho v = z$. Now we assert that $\mathbb{M}(\psi v, z, t) = 1$, $\mathbb{N}(\psi v, z, t) = 0$. If not then using equation (1) we have

$$\int_0^{\mathbb{M}(\psi v, \mu x_n, t)} \zeta(t)dt \geq \int_0^{\phi(\min\{\mathbb{M}(\rho v, \xi y_n, t), \mathbb{M}(\psi v, \rho v, t), \mathbb{M}(\mu y_n, \xi y_n, t), \mathbb{M}(\psi v, \xi y_n, t), \mathbb{M}(\rho v, \mu y_n, t)\})} \zeta(t)dt$$

for all $n \in$ N which on letting $n \to \infty$ and making use of lemma 1.1 give rise

$$\int_0^{\mathbb{M}(z, \psi v, t)} \zeta(t)dt \geq \int_0^{\phi(\mathbb{M}(z, \psi v, t))} \zeta(t)dt$$

$$\int_0^{\mathbb{N}(z, \psi v, t)} \zeta(t)dt \leq \int_0^{\Psi(\mathbb{N}(z, \psi v, t))} \zeta(t)dt$$

Since $z \neq \psi v$, therefore $0 < \mathbb{M}(z, \psi v, t) < 1$ for some $t_0 > 0$. As $\mathbb{M}(z, \psi v, t)$ is left continuous and $\mathbb{M}(z, \psi v, t)$ is nondecreasing, it can assume at the most countable points of discontinuity. If we assume that $t_0$ is a continuity point of $\mathbb{M}(z, \psi v, .)$then in view of condition (c), one gets $\phi(\mathbb{M}(z, \psi v, t_0)) > M(z, \psi v, t_0)$ and $\Psi(\mathbb{N}(z, \psi v, t_0)) < N(z, \psi v, t_0)$ which is a contradiction. Therefore $z = \psi v$ so that $\psi v = z = \xi v$. This shows that $v$ is a coincidence point of the pair $(\psi, \xi)$.

As $\xi(X)$ is a closed subset of $X$, therefore $\lim_{n\to\infty} \xi y_n = z \in \xi(x)$ . Also there is a point $s \in X$ such that $\xi s = z$. Now we assert that $\mathbb{M}(gs, z, t) = 1$. If not, then using condition (1) we have

$$\int_0^{\mathbb{M}(\psi v, \mu s, t_0)} \zeta(t)dt \geq \int_0^{\phi(\min\{\mathbb{M}(\rho v, \xi s, t), \mathbb{M}(\psi v, \rho s, t), \mathbb{M}(\mu v, \xi v, t), \mathbb{M}(\psi v, \xi v, t), \mathbb{M}(\rho v, \mu s, t)\})} \zeta(t)dt$$

$$\int_0^{\mathbb{M}(z, \mu s, t_0)} \zeta(t)dt \geq \int_0^{\phi(\min\{\mathbb{M}(z, \mu s, t)\})} \zeta(t)dt$$

$$\int_0^{\mathbb{N}(\psi v, \mu s, t_0)} \zeta(t)dt \leq \int_0^{\Psi(\max\{\mathbb{N}(\rho v, \xi s, t), \mathbb{N}(\psi v, \rho s, t), \mathbb{N}(\mu v, \xi v, t), \mathbb{N}(\psi v, \xi v, t), \mathbb{N}(\rho v, \mu s, t)\})} \zeta(t)dt$$

$$\int_0^{\mathbb{N}(z, \mu s, t_0)} \zeta(t)dt \leq \int_0^{\Psi(\max\{\mathbb{N}(z, \mu s, t)\})} \zeta(t)dt$$

As $z \neq \mu s$, then $0 < \mathbb{M}(z, \mu s, t_0) < 1$ for some $t_0$. As $\mathbb{M}(z, \mu s, .)$ is left continuous and $\mathbb{M}(z, \mu s, .)$ is nondecreasing, it has only (atmost) countable points of discontinuity. Similarly $0 < \mathbb{N}(z, \mu s, t_0) < 1$ for some $t_0$ As $\mathbb{N}(z, \mu s, .)$ is right continuous and the $\mathbb{N}(z, \mu s, .)$is nonincreasing and it has only countable point of discontinuity. Now one may suppose that $t_0$ is a continuity point of $\mathbb{M}(z, \mu s, .)$ and $\mathbb{N}(z, \mu s, .)$then according to condition (b) $\phi(\mathbb{M}(z, \mu s, t_0)) > M(z, \mu s, t_0)$ and $\Psi(\mathbb{N}(z, \mu s, t_0)) < N(z, \mu s, t_0)$ which is a contradiction. Therefore $z = \mu s$, therefore $\mu s = z = \xi s$. Hence s is a coincidence point of the pair $(\mu, \xi)$

Since $\psi v = \rho s$ and the pair $(\psi, \rho)$ is weakly compatible, therefore $\psi z = \psi \rho v = \rho \psi v = \rho z$. Now we need to show that $z$ is common fixed point of the pair $(\psi, \rho)$. To fullfill this we assume that $\mathbb{M}(\psi z, z, t) = 1$. If not, then using condition (1) we have

$$\int_0^{\mathbb{M}(\psi z,\mu s,t)} \zeta(t)dt \geq \int_0^{\phi(min\{\mathbb{M}(\rho z,\xi s,t),\mathbb{M}(\psi z,\rho z,t),\mathbb{M}(\mu s,\xi s,t),\mathbb{M}(\psi z,\xi s,t),\mathbb{M}(\rho z,\mu s,t)\})} \zeta(t)dt$$

$$\int_0^{\mathbb{M}(\psi z,z,t)} \zeta(t)dt \geq \int_0^{\phi(min\{\mathbb{M}(\rho z,z,t),1,1,\mathbb{M}(\psi z,z,t),\mathbb{M}(\psi z,z,t)\})} \zeta(t)dt$$

$$\int_0^{\mathbb{N}(\psi z,\mu s,t)} \zeta(t)dt \leq \int_0^{\Psi(max\{\mathbb{N}(\rho z,\xi s,t),\mathbb{N}(\psi z,\rho z,t),\mathbb{N}(\mu s,\xi s,t),\mathbb{N}(\psi z,\xi s,t),\mathbb{N}(\rho z,\mu s,t)\})} \zeta(t)dt$$

$$\int_0^{\mathbb{N}(\psi z,z,t)} \zeta(t)dt \leq \int_0^{\Psi(max\{\mathbb{N}(\rho z,z,t),0,0,\mathbb{N}(\psi z,z,t),\mathbb{N}(\psi z,z,t)\})} \zeta(t)dt$$

then $\psi z \neq z$, then $0 < \mathbb{M}(\psi z, z, t_0) < 1$ for some $t_0 > 0$. As $\mathbb{M}(\psi z, z, .)$ is left continuous and $\mathbb{M}(\psi z, z, .)$ is nondecreasing, it has only (atmost) countable points of discontinuity. Similarly $0 < \mathbb{N}(z, \mu s, t_0) < 1$ for some $t_0$ As $\mathbb{N}(\psi z, z, .)$ is right continuous and the $\mathbb{N}(\psi z, z, .)$is nonincreasing and it has only countable point of discontinuity. Now one may suppose that $t_0$ is a continuity point of $\mathbb{M}(z, \mu s, .)$ and $\mathbb{N}(z, \mu s, .)$then according to condition (b).$\phi(\mathbb{M}(\psi z, z, t_0)) > M(\psi z, z, t_0)$ and $\Psi(\mathbb{N}(\psi z, z, t_0)) < N(\psi z, z, t_0)$ which is a contradiction. Therefore $\mathbb{M}(\psi z, z, t) = 1$ , $\mathbb{N}(\psi z, z, t) = 0$ so that $\psi z = z$therefore $z$ is a common fixed of $(\psi, \rho)$.

As $\mu s = \xi s$ and the pair $(\mu, \xi)$ is weakly compatible, therefore $\mu z = \mu \xi s = \xi \mu s = \xi s$. Next we show that $z$ is a common fixed point of the pair $(\mu, \xi)$. To do this, we assert that $\mathbb{M}(\mu z, z, t) = 1$. If not then using equation (1), we have

$$\int_0^{\mathbb{M}(\psi u,\mu z,t)} \zeta(t)dt \geq \int_0^{\phi(min\{\mathbb{M}(\rho v,\xi z,t),\mathbb{M}(\psi v,\rho s,t),\mathbb{M}(\mu z,\xi z,t),\mathbb{M}(\psi u,\xi z,t),\mathbb{M}(\rho v,\mu z,t)\})} \zeta(t)dt$$

$$\int_0^{\mathbb{M}(\mu z,z,t)} \zeta(t)dt \geq \int_0^{\phi(min\{\mathbb{M}(\mu z,z,t),1,1,\mathbb{M}(\psi z,z,t),\mathbb{M}(\psi z,z,t)\})} \zeta(t)dt = \int_0^{\phi(\mathbb{M}(\mu z,z,t))} \zeta(t)dt$$

As earlier, one obtain $\mu z = z$ which shows that $z$ is a common fixed point of the pair $(\mu z, \xi)$. Hence $z$ is a common fixed point of $\psi$, $\mu$, $\rho$ and $\xi$. Uniqueness of the common fixed point is an easy consequences of equation (1) .

**Remark 2.1**Theorem 2.1 never requires conditions on the continuity of the involved mappings, completeness of the space and containment amongst range sets of involved mappings besides noted improvements in commutativity requirements.

**Theorem 2.2** The conclusion of Theorem 2.1 remain true if the condition (II) (of Theorem 2.1) is replaced by the following

(II) $\overline{\psi(X)} \subset \xi(X)$ and $\overline{\mu(X)} \subset \rho(X)$

Corollary: The conclusionof Theorems 2.1 and 2.2 remains true if the conditions (II) and (II)' are replaced by the following:

(II)'' $\psi(X)$ and $\mu(X)$ are closed subsets of $X$provided $\psi(X) \subset \xi(X)$ and $\mu(X) \subset \rho(X)$.

**Theorem 2.3** Let $\psi, \mu$, ρ and $\xi$ be four self mapping of IFMS $(X, \mathbb{M}, \mathbb{N}, *, \diamond)$ fulfill the following axioms from (1)-(4) of Lemma 2.1. Suppose that

(5) $\rho(X)$ (or $\xi(X)$) is a closed subset of $X$.

Then the pair $(\psi, \rho)$ as well as $(\mu, \xi)$have a coincidence point. Moreover $\psi, \mu$, ρ and $\xi$have a unique common fixed point in $X$provided the pairs $(\psi, \rho)$ and $(\mu, \xi)$ are weakly compatible.

Proof: Since the pairs $(\psi, \rho)$ and $(\mu, \xi)$share the common property (E.A.) therefore the exists two sequences $\{x_n\}$ and $\{y_n\}$ in $X$ such that

$\lim_{n\to\infty} \psi x_n = \lim_{n\to\infty} \rho x_n = \lim_{n\to\infty} \mu y_n = \lim_{n\to\infty} \xi y_n = z$ for some $z \in X$

Since $\rho(X)$ is a closed subset of $X$, On the lines of the proof of Theorem 2.1 we can show that $(\psi, \rho)$ has a coincidence point, say $v$ i.e. $\psi v = z = \rho v$. since $\psi(X) \subset \xi(X)$ and $\psi v \in \psi(X)$, there exists $s \in X$ such that $z = \psi v = \xi s$. Now we assert that $\mathbb{M}(gs, z, t) = 1.\mathbb{N}(gs, z, t) = 0$ If not,then using equation (1) we have

$$\int_0^{\mathbb{M}(\psi v,\mu s,t)} \zeta(t)dt \geq \int_0^{\phi(min\{\mathbb{M}(\rho v,\xi s,t),\mathbb{M}(\psi v,\rho v,t),\mathbb{M}(\mu s,\xi s,t),\mathbb{M}(\psi v,\xi s,t),\mathbb{M}(\rho v,\mu s,t)\})} \zeta(t)dt$$

$$\int_0^{\mathbb{M}(z,\mu s,t)} \zeta(t)dt \geq \int_0^{\phi(min\{1,\mathbb{M}(\mu s,z,t),1,1,\mathbb{M}(z,\mu s,t),\})} \zeta(t)dt = \int_0^{\phi(\mathbb{M}(z,\mu s,t))} \zeta(t)dt$$

$$\int_0^{\mathbb{N}(\psi v,\mu s,t)} \zeta(t)dt \leq \int_0^{\Psi(max\{\mathbb{N}(\rho v,\xi s,t),\mathbb{N}(\psi v,\rho v,t),\mathbb{N}(\mu s,\xi s,t),\mathbb{N}(\psi v,\xi s,t),\mathbb{N}(\rho v,\mu s,t)\})} \zeta(t)dt$$

$$\int_0^{\mathbb{N}(z,\mu s,t)} \zeta(t)dt \leq \int_0^{\Psi(max\{0,\mathbb{N}(\mu s,z,t),0,0,\mathbb{N}(z,\mu s,t),\})} \zeta(t)dt = \int_0^{\Psi(\mathbb{N}(z,\mu s,t))} \zeta(t)dt$$

Applying the analogous arguments as in Theorem 2.1 we can easily show that $\mu s = z = \xi s$. The rest of the proof can be completed on the lines of Theorem 2.1 This completes the proof.

**Corollary2.1** Let $\psi$ and $\rho$ be two self mapping of IFMS $(X, \mathbb{M}, \mathbb{N},*,\diamond)$ satisfying the following conditions:

(1) The pair $(\psi, \rho)$ satisfy the property (E.A.),

(2) $\rho(X)$ is a closed subset of $X$,

(3) $\forall x, y \in X, x \neq y, \exists t > 0$: $0 < \mathbb{M}(x, y, t) < 1$ and $0 < \mathbb{N}(x, y, t) < 1$ and for some $\phi \in \Phi$, $\Psi \in \Phi$

$$\int_0^{\mathbb{M}(\psi x,\psi y,t)} \zeta(t)dt \geq \int_0^{\phi(min\{\mathbb{M}(\rho s,\rho y,t),\mathbb{M}(\psi x,\rho x,t),\mathbb{M}(\psi y,\rho y,t),\mathbb{M}(\psi x,\rho y,t),\mathbb{M}(\rho x,\psi y,t)\})} \zeta(t)dt$$

$$\int_0^{\mathbb{N}(\psi x,\psi y,t)} \zeta(t)dt \leq \int_0^{\Psi(max\{\mathbb{N}(\rho s,\rho y,t),\mathbb{N}(\psi x,\rho x,t),\mathbb{N}(\psi y,\rho y,t),\mathbb{N}(\psi x,\rho y,t),\mathbb{N}(\rho x,\psi y,t)\})} \zeta(t)dt$$

Then pair $(\psi, \rho)$has a coincidence point. Moreover $\psi$and $\rho$have a unique common fixed point in $X$provided that the pair $(\psi, \rho)$is weakly compatible.

**Theorem: 2.4** Let $\{\psi_1, \psi_2, \ldots, \psi_m\}$, $\{\mu_1, \mu_2, \ldots, \mu_p\}$,$\{\rho_1, \rho_2, \ldots, \rho_n\}$ and $\{\xi_1, \xi_2, \ldots, \xi_q\}$ be four finite families of self mapping of IFMS$(X, \mathbb{M}, \mathbb{N},*,\diamond)$ with $\psi = \psi_1\psi_2 \ldots \psi_m$, $\mu = \mu_1\mu_2, \ldots, \mu_p$, $\rho = \rho_1\rho_2, \ldots, \rho_n$ and $\xi = \xi_1\xi_2, \ldots, \xi_q$ satisfying equation (1) and pairs $(\psi, \rho)$ and $(\mu, \xi)$share the common property (E.A.). If $\rho(X)$ and $\xi(X)$ are closed subset of $X$, then

(i) the pair $(\psi, \rho)$ and $(\mu, \xi)$have a coincidence point each.

Moreover $\psi_i$,$\rho_k$, $\mu_r$ and $\xi_t$have a unique common fixed point provided the pairs of families $(\{\psi_i\}, \{\rho_k\})$ and $(\{\mu_r, \{\xi_t\}\})$commute pairwise, where $i \in \{1,2, \ldots, m\}$, $k \in \{1,2, \ldots, n\}$, $r \in \{1,2, \ldots, p\}$ and $t \in \{1,2, \ldots, q\}$

proof: proof follows on the lines of the corresponding result contained in 2.1

By setting $\psi_1 = \psi_2 = \cdots = \psi_m = \mathrm{A}$, $\mu_1 = \mu_2 = \cdots = \mu_p = \mathrm{B}$, $\rho_1 = \rho_2 = \cdots = \rho_n = \mathrm{C}\xi_1 = \xi_2 = \cdots = \xi_q = \mathrm{D}$ in Theorem 2.4, we deduce the following result for iterates of mappings:

**Corollary 2.2Let** A, B, C and D be four self mapping of IFMS $(X, \mathbb{M}, \mathbb{N},*,\diamond)$, pairs $(\mathrm{A}^m, \mathrm{C}^n)$ and $(\mathrm{B}^p, \mathrm{D}^q)$share the common property (E.A.) and satisfying the condition $\forall x, y \in X$, $x \neq y$, $\exists t > 0$: $0 < \mathbb{M}(x, y, t) < 1$ for some $\phi \in \Phi$, $\Psi \in \Phi$.

$$\int_0^{\mathbb{M}(A^m x,B^m y,t)} \zeta(t)dt \geq \int_0^{\phi(min\{\mathbb{M}(C^n x,D^q y,t),\mathbb{M}(A^m x,C^n x,t),\mathbb{M}(B^p y,D^q y,t),\mathbb{M}(A^m x,D^q y,t),\mathbb{M}(C^n x,B^p y,t)\})} \zeta(t)dt$$

$$\int_0^{\mathbb{N}(A^m x,B^m y,t)} \zeta(t)dt \geq \int_0^{\Psi(max\{\mathbb{N}(C^n x,D^q y,t),\mathbb{N}(A^m x,C^n x,t),\mathbb{N}(B^p y,D^q y,t),\mathbb{N}(A^m x,D^q y,t),\mathbb{N}(C^n x,B^p y,t)\})} \zeta(t)dt$$

where $m, n, p, q \in \mathrm{Z}^+$. If $C^n(X)$ and $D^q(X)$ are closed subset of $X$, then $A, B, C, D$have a unique common fixed point provided the pair $(A, C)$ as well as $(B, D)$is commuting.

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