Finite Integrals Involving Multivariable H-Function with The Product of M-Series and Polynomials

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Abstract - In this research paper the author documented the integrals involving the result of general class of polynomials and find the solution of finite integral involving the product of multivariable H-function and M-series, the M-series is a particular case of the $\bar{H}$-function of Innayat-Hussain [3] these results were derived for basic in nature and large number of integrals involving simpler and polynomials as their particular cases. These results were important in various application of H-function.

Keywords: Multivariable Polynomials, Multivariable of H-function, M-series, Beta function.

I. INTRODUCTION

Recently the integral involving general class of polynomial with H-function are evaluated in the research paper. We established the results of the integrals involving Multivariable H-function with the product of the M-series and polynomials, Also we discussed some special cases of multivariable H-function and M-series.

Special type of the H-function of one variable is defined Mukhergee and Prasad [5]

$$H^{m'+1,n'}_{p',q'+1}\left[\left(a_0\left(az+\frac{b}{z}\right)^2+d\right)^{-\gamma}\Big|^{(r_k,R_k)1,p}_{(l_0,L_0),(l_k,L_k)1,q}\right]=\frac{1}{L_0}\sum_{s=0}^{\infty}M(s)\left(a_0\left(az+\frac{b}{z}\right)^2+d\right)^{-\gamma}\rho_s \tag{1.1}$$

where

$$M(s)=\frac{\prod_{k=1}^{m'}\Gamma(l_k-L_k\rho_s)\prod_{k=1}^{n'}\Gamma(1-r_k+R_k\rho_s)}{\prod_{k=m'+1}^{q'}\Gamma(1-l_k+L_k\rho_s)\prod_{k=n'+1}^{p'}\Gamma(r_k-R_k\rho_s)}\frac{(-1)^r}{r!}\left[a_0\left(az+\frac{b}{z}\right)^2+d\right]\rho_s \tag{1.1a}$$

Provided(1). $\left(\Gamma_{i=1}^{s}\left(1-c_{ik}^{(i)}+\gamma_{ik}^{(i)}\lambda_i\right)\right)\geq 0, L\leq \operatorname{Re}\left(\frac{l_0}{L_0}\right)\leq\delta$,(II)|arg r | < 1/2π,| argr |< $\frac{1}{2}\lambda\pi,(\lambda R>)$

$$\lambda=\sum_{k=1}^{n'}R_k-\sum_{k=1}^{n'}R_k+\sum_{k=1}^{m'}L_k-\sum_{k=m+1}^{q'}L_k\ ,\ R=\sum_{k=1}^{q'}L_k-\lambda=\sum_{k=1}^{p'}R_k\ ,\ \rho_s=\frac{l_0+S}{L_0}$$

The multivariable H-function introduced by Srivastava and Panda [13]

$$H^{o,n:m_1,n_1;..;m_s,n_s}_{p,q:p_1q_1;....;p_s,q_s}\left[\begin{matrix}z_1\\ \vdots\\ z_s\end{matrix}\middle|\begin{matrix}(a_k:\alpha_{1k'};...;\alpha_{sk})_{1,p}:(c_{1k},\gamma_{1k})1,p_1;...;(c_{sk},\gamma_{sk})_{1,P_s}\\ (b_k:\beta_{1k'}...;\beta_{sk})_{1,q}:(d_{1k'}\delta_{1k})1,q_1;...;(d_{sk},\delta_{sk})_{1,q_s}\end{matrix}\right]$$

$$=\frac{1}{(2\pi w)^s}\int_{\xi_1}..\int_{\xi_s}\theta(\lambda_1,...,\lambda_s)\phi_1(\lambda_1)..\phi_s(\lambda_s)z_1^{\lambda_1},...,z_s^{\lambda_s}d\lambda_1,...,d\lambda_s\ , \tag{1.2}$$

where

$$\theta(\lambda_1,...,\lambda_s)=\frac{\prod_{k=1}^{n}\Gamma_{i=1}^{s}\left(1-a_k+\sum_{i=1}^{s}\alpha_{ik}\lambda_i\right)}{\prod_{k=1}^{q}\Gamma_{i=1}^{s}\left(1-b_k+\sum_{i=1}^{s}\beta_{ik}\lambda_i\right)\prod_{k=n+1}^{p}\Gamma_{i=1}^{s}\left(a_k-\sum_{i=1}^{s}\alpha_{ik}\lambda_i\right)},\quad \text{and} \tag{1.2a}$$

$$\phi_i(\lambda_i) = \frac{\prod_{k=1}^{m_i} \Gamma_{i=1}^{s}\left(d_{ik} - \delta_{ik}\lambda_i\right) \prod_{k=1}^{n_i} \Gamma_{i=1}^{s}\left(1 - c_{ik} + \gamma_{ik}\lambda_i\right)}{\prod_{k=m_i+1}^{q_i} \Gamma_{i=1}^{s}\left(1 - d_{ik} + \delta_{ik}\lambda_i\right) \prod_{k=n_i+1}^{p_i} \Gamma_{i=1}^{s}\left(c_{ik} - \gamma_{ik}\lambda_i\right)}, \qquad (1.2b)$$

Where n, p, q, m_i, n_i, p_i, q_i, i are none- negative integer. Such that $0 \le n \le p, q \ge 0,\ 0 \le m_i \le q_i$ and $0 \le n_i \le p_i, i = 1,\ldots s$ .and $\alpha_{ik}, \beta_{ik}, \gamma_{ik}\, and\, \delta_{ik}^{i}$ are all positive. The contour ξ_i lies in the complex plane λ_i is of mellin-Barnes type contours integral which start at the point $\tau_i - \omega_\infty$ and go $\tau_i + \omega_\infty$ with τ_i €(-∞,∞). Such that all the poles of $\Gamma_{i=1}^{s}\left(d_{ik} - \gamma_{ik}\lambda_i\right); i = 1,2,\ldots s$ and $\Gamma_{i=1}^{s}\left(1 - a_{ik} + \gamma_{ik}\lambda_i\right); k = 1.2\ldots.n$ are to the left of ξ_i .

$$\nabla_i = \sum_{k=1}^{p} \alpha_{ik} + \sum_{k=1}^{p_i} \gamma_{ik} - \sum_{k=1}^{q} \beta_{ik} - \sum_{k=1}^{q_i} \delta_{ik} \le 0 \qquad (1.2c)$$

$$\Re_i = \sum_{k=1}^{n} \alpha_{ik} - \sum_{k=n+1}^{p} \alpha_{ik} + \sum_{k=1}^{q} \beta_{ik} + \sum_{k=1}^{n_i} \gamma_{ik} - \sum_{k=n_i+1}^{p_i} \gamma_{ik} + \sum_{k=1}^{m_i} \delta_{ik} - \sum_{k=m_i+1}^{q_i} \delta_{ik} > 0 \qquad (1.2d)$$

It is assumed that the poles of of integrand of (1.2) are simpler under the condition (1.2d) the integrals in (1.2) converge absolutely [13] in the domain.

$$\left|\arg(k_i)\right| \frac{\pi}{2} < \Re_i,\ , i = 1,\ldots s$$

The M-series is particular case of the H-function of Innayat-Hussain [3]. It is special role in the application of fractional calculus operators and in the solution of multivariable H-function and polynomials. Manoj Sharma [9] given the definition of M-series.

$${}_{P}M_{Q}^{\varphi}[Z] = \sum_{s'=0}^{\infty} \frac{\left(a_1\right)_{s'};\ldots;\left(a_P\right)_{s'}}{\left(b_1\right)_{s'};\ldots;\left(b_Q\right)_{s'}} \frac{Z^{s'}}{\Gamma(\varphi s'+1)} \qquad (1.3)$$

$$\text{L(s')} = \frac{\prod_{l=1}^{P} (a_l)_{s'}}{\prod_{l=1}^{Q} (b_l)_{s'}} \frac{Z^{s'}}{\Gamma(\varphi s'+1)} \qquad (1.3a)$$

Here $\varphi \epsilon$c, R(φ)>0 and $(a_l)_{s'}$, $(b_l)_{s'}$ are pochammer's symbols. The series [1.3] is defined when none of the denominator parameter $(b_l)_{s'}$ l=1,2,…Q is a negative integer or zero. Series is interesting because this is a particular cases of ${}_{P}F_{Q}$ hypergeometric function [9] and the Mittag-Leffter function [4] [L-1] when φ=1, in equation (1.3) then the series ${}_{P}M_{Q}^{\varphi}[Z]$ becomes the generalized hypergeometric function see[9]

$${}_{P}M_{Q}^{1}[Z] = \sum_{s'=0}^{\infty} \frac{\left(a_1\right)_{s'};\ldots;\left(a_P\right)_{s'}}{\left(b_1\right)_{s'};\ldots;\left(b_Q\right)_{s'}} \frac{Z^{s'}}{\Gamma(s'+1)} = {}_{P}F_{Q} \qquad (1.3b)$$

$$\text{G(s')} = \frac{\prod_{l=1}^{P} (a_l)_{s'}}{\prod_{l=1}^{Q} (b_l)_{s'}} \frac{Z^{s'}}{\Gamma(s'+1)} \qquad (1.3c)$$

[L-2] when in equation (1.3) there is no lower and upper parameters (${}_0M_0^1[z]$) or (P=Q=0) then the series is change to the Mittag-Leffter function [4]

$$ {}_0M_0^{\varphi}[...;...;z] = \sum_{s'=0}^{\infty} \frac{Z^{s'}}{\Gamma(\varphi s'+1)} = E_{\varphi'}[z] \tag{1.3d} $$

$$ \mathrm{T(s')} = \frac{Z^{s'}}{\Gamma(\varphi s'+1)} \tag{1.3e} $$

First class of multivariable polynomial given by Srivastava and Garg [12]

$$ S_n^{m_1,\dots,m_k}[y_1 ... y_k] = \sum_{r_1,\dots,r_k=0}^{m_1r_1+\dots m_kr_k \le n} (-n)_{m_1r_1+\dots+m_kr_k}\ A(n_1;r_1,\dots,r_k)\ \frac{y_1^{r_1}}{r_1!} \cdots \frac{y_k^{r_k}}{r_k!}\,, \tag{1.4} $$

The second class of multivariable polynomials introduced by Srivastava [16] is defined as follows.

$$ S_{n_1,\dots,n_k}^{m_1,\dots,m_k}[y_1,\dots,y_k] = \sum_{r_1=0}^{[n_1/m_1]},\dots,\sum_{r_k=0}^{[n_k/m_k]} (-n_1)_{m_1r_1},\dots,(-n_k)_{m_kr_k} $$

$$ A(n_1,r_1;\dots;n_k,r_k)\ \frac{y_1^{r_1}}{r_1!},\dots,\frac{y_k^{r_k}}{r_k!} \tag{1.5} $$

Table integrals formulae given by Gradshteyin and Ryzhik [2]

$$ \int_{-1}^{1} (1-\mu)^u (1+\mu)^v d\mu = 2^{u+v+1} B\,(u+1,\ v+1), \tag{1.6} $$

$$ B(u,v) = \frac{\Gamma(u)\Gamma(v)}{\Gamma(u+v)} \tag{1.7} $$

$$ \int_0^{\infty} \left[\left(az+\frac{b}{z}\right)^2 + d\right]^{-u-1} dz = \frac{\sqrt{\pi}\,\Gamma(u+½)}{2a(4ab+c)^{u+½}\,\Gamma(u+1)} \quad \mathbf{Re}(u)+½>0 \tag{1.8} $$

II. RESULT AND DISCUSSION

First integral

$$ \int_{-1}^{1}(1-\mu)^{\rho}(1+\mu)^{\sigma} S_{n_1,\dots,n_k}^{m_1,\dots,m_k}\left[y_1(1-\mu)^{m_1}(1+\mu)^{n_1},\dots,y_k(1-\mu)^{m_k}(1+\mu)^{n_k}\right] {}_PM_Q^{\varphi}[Z(1-\mu)^u(1+\mu)^v] \times $$

$$ H_{p,q:p_1q_1;\dots;p_s,q_s}^{o,n:m_1,n_1;..;m_s,n_s}\left[\begin{matrix} z_1(1-\mu)^{g_1}(1+\mu)^{h_1}, \\ \vdots \\ z_s(1-\mu)^{g_s}(1+\mu)^{h_s}, \end{matrix}\left|\begin{matrix} (a_k:\alpha_{1k'};\dots;\alpha_{sk})_{1,p}:(c_{1k},\gamma_{1k})1,p_1;\dots;(c_{sk},\gamma_{sk})_{1,P_s} \\ (b_k:\beta_{1k'}\dots;\beta_{sk})_{1,q}:(d_{1k'}\,\delta_{1k})1,q_1;\dots;(d_{sk},\delta_{sk})_{1,q_s} \end{matrix}\right.\right] d\mu = $$

$$ 2^{\rho+\sigma+1}\sum_{r_1=0}^{[n_1/m_1]},\dots,\sum_{r_k=0}^{[n_k/m_k]}(-n_1)_{m_1r_1},\dots,(-n_k)_{m_kr_k}\ A(n_1,r_1;\dots;n_k,r_k)\ \prod_{i=1}^{k}\frac{y_i^{r_i}}{r_i!}\ 2^{(u+v)s'}\sum_{s'}^{\infty}L(s') $$

$$ 2^{\sum_{i=1}^{s}(m_i+n_i)r_i}\ H_{p+2,q+1}^{0,n+2}\left[\begin{matrix} z_1 2^{(g_1+h_1)\lambda_1} \\ \vdots \\ z_s 2^{(g_s+h_s)\lambda_s} \end{matrix}\left|\begin{matrix} \left(-\sigma-us'-\sum_{i=1}^{k}(m_ir_i),;g_{1,\dots}g_s\right),\left(-\rho-vs'-\sum_{i=1}^{k}(n_ir_i),;h_{1,\dots}h_s\right),(a_j:\alpha_{1k};\dots;\alpha_{sk})_{1,p}:(c_{1k},\gamma_{1k})1,p_1;\dots;(c_{sk},\gamma_{sk})_{1,P_s} \\ (b_j:\beta_{1k};\dots;\beta_{sk})_{1,q},\left(-\rho-\sigma-(u+v)s'-\sum_{i=1}^{k}(m_i+n_i)r_i-1:g_1+h_1;\dots;g_s+h_s\right):(d_{1k'}\,\delta_{1k})1,q_1;\dots;(d_{sk},\delta_{sk})_{1,q_s} \end{matrix}\right.\right] $$

(2.1)

where m_i>0 (i=1, …s), n_i>0 (i=1,..,s), h>,0,g>,0 and g+h>0(not both are zero simultaneously).

proof : Taking L.H.S of (2.1)

$$\int_{-1}^{1}(1-\mu)^{\rho}(1+\mu)^{\sigma}S_{n_1,\dots,n_k}^{m_1,\dots,m_k}\left[y_1(1-\mu)^{m_1}(1+\mu)^{n_1},\dots,y_k(1-\mu)^{m_k}(1+\mu)^{n_k}\right]{}_PM_Q^{\varphi}[Z(1-\mu)^{u}(1+\mu)^{v}]\times$$

$$H_{p,q:p_1q_1;\dots;p_s,q_s}^{o,n:m_1,n_1;..;m_s,n_s}\left[\begin{matrix}z_1(1-\mu)^{g_1}(1+\mu)^{h_1}\\ \vdots\\ z_s(1-\mu)^{g_s}(1+\mu)^{h_s}\end{matrix}\left|\begin{matrix}(a_k:\alpha_{1k};\dots;\alpha_{sk})_{1,p}:(c_{1k},\gamma_{1k})_{1,p_1};\dots;(c_{sk},\gamma_{sk})_{1,P_s}\\ (b_k:\beta_{1k},\dots;\beta_{sk})_{1,q}:(d_{1k},\delta_{1k})_{1,q_1};\dots;(d_{sk},\delta_{sk})_{1,q_s}\end{matrix}\right.\right]d\mu$$

using (1.2), (1.3) and (1.5) and interchanging integration and summation

$$\sum_{r_1=0}^{[n_1/m_1]},\dots,\sum_{r_k=0}^{[n_k/m_k]}\sum_{s'=0}^{\infty}L(s')(-n_1)_{m_1r_1},\dots,(-n_k)_{m_kr_k}A(n_1,r_1;\dots;n_k,r_k)\prod_{i=1}^{k}\frac{y_i^{r_i}}{r_i!}\times$$

$$\frac{1}{(2\pi w)^s}\int_{\xi_1}\cdot\cdot\int_{\xi_r}\theta_i(\lambda_i,\dots,\lambda_s)\phi_1(\lambda_i)\dots\phi_s(\lambda_s)z_1^{\lambda_i},\dots,z_s^{\lambda_s}d\lambda_i,\dots,d\lambda_s\times$$

$$\int_{-1}^{1}(1-\mu)^{\rho+us'+\sum_{i=1}^{k}m_ir_i+\sum_{i=1}^{s}g_i\lambda_i}(1+\mu)^{\sigma+vs'+\sum_{i=1}^{k}n_ir_i+\sum_{i=1}^{s}h_i\lambda_i}d\mu.$$

Thus we evaluate the integral of the above equation with the help of integral (1.6) and solve the multivariable H-function on the basis of the definition of contour integral.

$$\sum_{r_1=0}^{[n_1/m_1]},\dots,\sum_{r_k=0}^{[n_k/m_k]}\sum_{s'=0}^{\infty}L(s')(-n_1)_{m_1r_1},\dots,(-n_k)_{m_kr_k}A(n_1,r_1;\dots;n_k,r_k)\prod_{i=1}^{k}\frac{y_i^{r_i}}{r_i!}\times$$

$$2^{\rho+\sigma+(u+v)s'\sum_{i=1}^{k}(m_i+n_i)r_i+\sum_{i=1}^{s}(g_i+h_i)\lambda_i+1}$$

$$\frac{1}{(2\pi w)^s}\int_{\xi_1}\dots\int_{\xi_r}\phi_1(\lambda_1)\dots\phi_s(\lambda_s)\frac{\Gamma\left(1-(-\rho+us'-\sum_{i=1}^{k}m_ir_i+\sum_{i=1}^{s}g_i\lambda_i\right)\Gamma\left(1-(-\sigma+vs'-\sum_{i=1}^{k}n_ir_i+\sum_{i=1}^{s}h_i\lambda_i\right)\prod_{k=1}^{n}\Gamma_{i=1}^{s}\left(1-a_k+\sum_{i=1}^{s}\alpha_{ik}\lambda_i\right)}{\prod_{k=1}^{q}\Gamma_{i=1}^{s}\left(1-b_k+\sum_{i=1}^{s}\beta_{ik}\lambda_i\right)\Gamma\left(1-(-\rho-\sigma-(u+v)s'\sum_{i=1}^{k}(m_i+n_i)r_i-1)+\sum_{i=1}^{s}(g_i+h_i)\lambda_i\right)}$$

$$z_1^{\lambda_i},\dots,z_s^{\lambda_s}d\lambda_1,\dots,d\lambda_s$$

Using (1.2) in above equation, the right side of (2.1) will be found.

where $m_i>0$ (i=1,…,s) $n_i>0$ (i=1…..s) h>0; g>,0.

Second integral

$$\int_{-1}^{1}(1-\mu)^{\rho}(1+\mu)^{\sigma}S_{n}^{m_1,\dots,m_k}\left[y_1(1-\mu)^{m_1}(1+\mu)^{n_1},\dots,y_k(1-\mu)^{m_k}(1+\mu)^{n_k}\right]{}_PM_Q^{\varphi}[Z(1-\mu)^{u}(1+\mu)^{v}]\times$$

$$H_{p,q:p_1q_1;\dots;p_s,q_s}^{o,n:m_1,n_1;..;m_s,n_s}\left[\begin{matrix}z_1(1-\mu)^{g_1}(1+\mu)^{h_1}\\ \vdots\\ z_s(1-\mu)^{g_s}(1+\mu)^{h_s}\end{matrix}\left|\begin{matrix}(a_k:\alpha_{1k};\dots;\alpha_{sk})_{1,p}:(c_{1k},\gamma_{1k})_{1,p_1};\dots;(c_{sk},\gamma_{sk})_{1,P_s}\\ (b_k:\beta_{1k},\dots;\beta_{sk})_{1,q}:(d_{1k},\delta_{1k})_{1,q_1};\dots;(d_{sk},\delta_{sk})_{1,q_s}\end{matrix}\right.\right]d\mu$$

$$=2^{\rho+\sigma+1}\sum_{r_1,\dots,r_k=0}^{m_1r_1+\dots m_kr_k\le n}(-n)_{m_1r_1+\dots+m_kr_k}A(n;r_1,\dots,r_k)\prod_{i=1}^{k}\frac{y_i^{r_i}}{r_i!}2^{(u+v)s'}\sum_{s'}^{\infty}L(s')2^{\sum_{i=1}^{s}(m_i+n_i)r_i}\times$$

$$H_{p+2,q+1}^{0,n+2}\left[\begin{matrix}z_12^{(g_1+h_1)\lambda_1}\\ \vdots\\ z_s2^{(g_s+h_s)\lambda_s}\end{matrix}\left|\begin{matrix}\left(-\sigma-us'-\sum_{i=1}^{k}(m_ir_i),;g_1,\dots g_s\right),\left(-\rho-vs'-\sum_{i=1}^{k}(n_ir_i),;h_1,\dots h_s\right),(a_j:\alpha_{1k};\dots;\alpha_{sk})_{1,p}:(c_{1k},\gamma_{1k})_{1,p_1};\dots;(c_{sk},\gamma_{sk})_{1,P_s}\\ (b_j:\beta_{1k};\dots;\beta_{sk})_{1,q},\left(-\rho-\sigma-(u+v)s'-\sum_{i=1}^{k}(m_i+n_i)r_i-1:g_1+h_1;\dots;g_s+h_s\right):(d_{1k},\delta_{1k})_{1,q_1};\dots;(d_{sk},\delta_{sk})_{1,q_s}\end{matrix}\right.\right]$$

(2.2)

Proof: to solve the above equation (2.2), instead of the second class of polynomials used in (2.1), we will solve the first type of polynomials in the same way by substituting the value of the first type of polynomial from equation (1.4) and get the right side of (2.2).

Third integral

$$\int_0^\infty \left[\left(az+\frac{b}{z}\right)^2+d\right]^{-\vartheta-1} S_{n_1,\dots,n_k}^{m_1,\dots,m_k}\left[y_1\left(az+\frac{b}{z}\right)^2+d\right]^{-m_1},\dots,\left[y_k\left(az+\frac{b}{z}\right)^2+d\right]^{-m_k}$$

$$\times H_{p',q'+1}^{m'+1,n'}\left[\left[a_0\left(az+\frac{b}{z}\right)^2+d\right]^{-\gamma}\Big|_{(l_0,L_0),(l_k,L_k)1,q}^{(r_k,R_k)1,p}\right] {}_P M_Q^{\varphi}\left[z\left(az+\frac{b}{z}\right)^2+d\right]^{\eta_g}$$

$$H\left[\left[z_1\left(az+\frac{b}{z}\right)^2+d\right]^{-s_1},\dots,\left[z_s\left(az+\frac{b}{z}\right)^2+d\right]^{-s_s}\right]dz \tag{2.3}$$

$$=\frac{\sqrt{\pi}\,a_0^{-\gamma}}{2aL_0(4ab+d)^{\upsilon+\gamma+\eta_G+\frac{1}{2}}}\sum_{r_1=0}^{[n_1/m_1]},\dots,\sum_{r_k=0}^{[n_k/m_k]}\sum_{s=0}^{\infty}\sum_{s'=0}^{\infty}(-n_1)_{m_1r_1},\dots,(-n_k)_{m_kr_k}A(n_1,r_1;\dots;n_k,r_k)\;M(s)\rho_s L(s')$$

$$\prod_{i=1}^{k}\frac{y_i^{r_i}(4ab+d)^{-\sum_{i=1}^{k}m_ir_i}}{r_i!}H_{p+1,q+1:D}^{0,n+1:A}\left[\begin{matrix}z_1(4ab+d)^{-s_1}\\ \vdots\\ z_s(4ab+d)^{-s_s}\end{matrix}\middle|\begin{matrix}\left(\tfrac{1}{2}-\vartheta-\gamma-\eta_g s'+\sum_{i=1}^{k}m_ik_i;s_1,\dots,s_s\right),E:V\\ \left(-\vartheta-\gamma-\eta_g s'+\sum_{i=1}^{k}m_ik_i;s_1,\dots,s_s\right),F:W\end{matrix}\right]$$

Where: E= $(a_k:\alpha_{1k'};\dots;\alpha_{sk})_{1,p}$, F= $(b_j:\beta_{1k};\dots;\beta_{sk})_{1,q}$

V= $(c_{1k},\gamma_{1k})1,p_1;\dots;(c_{sk},\gamma_{sk})_{1,P_S}$, W= $(d_{1k},\delta_{1k})1,q_1;\dots;(d_{sk},\delta_{sk})_{1,q_S}$

where $A=m_1,n_1;\dots;m_s,n_s$, $D=p_1,q_1;\dots;p_s,q_s$

Proof: Taking the left side of equation (2.3) and solving of results from (1.1), (1.2), (1.3), and (1.5) respectively, we get the following type of equation.

$$\frac{a_0^{-\gamma}}{L_0}\sum_{r_1=0}^{[n_1/m_1]},\dots,\sum_{r_k=0}^{[n_k/m_k]}\sum_{s=0}^{\infty}\sum_{s'=0}^{\infty}(-n_1)_{m_1r_1},\dots,(-n_k)_{m_kr_k}A(n_1,r_1;\dots;n_k,r_k)\;M(s)\rho_s L(s')$$

$$\prod_{i=1}^{k}\frac{y_i^{r_i}}{r_i!}\frac{1}{(2\pi w)^s}\int_{\xi_1}\dots\int_{\xi_s}\theta_i(\lambda_i,\dots,\lambda_s)\phi_1(\lambda_i)\dots\phi_s(\lambda_s)z_1^{\lambda_i},\dots,z_s^{\lambda_s}$$

$$\int_0^\infty\left[\left(az+\frac{b}{z}\right)^2+d\right]^{-(\vartheta+\gamma+\eta_g s'-\sum_{i=1}^{k}(m_i,r_i)+\sum_{i=1}^{s}s_i,\lambda_i)-1}dz\,d\lambda_1,\dots,d\lambda_s$$

Where $\lambda_i,\dots,\lambda_s$ is represented for the variables of aforementioned Mellin-Barnes type contour integral of H-function. Now we evaluate the z-integral with the help of (1.8) and solve the resultant contour integral in terms of the multivariable H-function and M-series. We get the results for the other side of equation (2.3).

III. SPECIAL CASES

Case- I: from the equation (1.3b)

$$\int_{-1}^{1}(1-\mu)^{\rho}(1+\mu)^{\sigma}S_{n_1,\dots,n_k}^{m_1,\dots,m_k}\left[y_1(1-\mu)^{m_1}(1+\mu)^{n_1},\dots,y_k(1-\mu)^{m_k}(1+\mu)^{n_k}\right]{}_P F_Q[z(1-\mu)^u(1+\mu)^v]$$

$$H_{p,q:p_1q_1;\dots;p_s,q_s}^{o,n:m_1,n_1;\dots;m_s,n_s}\left[\begin{matrix}z_1(1-\mu)^{g_1}(1+\mu)^{h_1}\\ \vdots\\ z_s(1-\mu)^{g_s}(1+\mu)^{h_s}\end{matrix}\middle|\begin{matrix}(a_k:\alpha_{1k'};\dots;\alpha_{sk})_{1,p}:(c_{1k},\gamma_{1k})1,p_1;\dots;(c_{sk},\gamma_{sk})_{1,P_S}\\ (b_k:\beta_{1k'},\dots;\beta_{sk})_{1,q}:(d_{1k},\delta_{1k})1,q_1;\dots;(d_{sk},\delta_{sk})_{1,q_S}\end{matrix}\right]d\mu$$

$$= 2^{\rho+\sigma+1} \sum_{r_1=0}^{[n_1/m_1]},...,\sum_{r_k=0}^{[n_k/m_k]} (-n_1)_{m_1r_1},...,(-n_k)_{m_kr_k} A(n_1,r_1;...;n_k,r_k) \prod_{i=1}^{k} \frac{y_i^{r_i}}{r_i!} 2^{(u+v)s'} \sum_{s'}^{\infty} G(s') 2^{\sum_{i=1}^{s}(m_i+n_i)r_i}$$

$$H_{p+2,q+1}^{0,n+2}\left[\begin{matrix} z_1 2^{(g_1+h_1)\lambda_1} \\ \vdots \\ z_s 2^{(g_s+h_s)\lambda_s} \end{matrix}\middle| \begin{matrix} \left(-\sigma-us'-\sum_{i=1}^{k}(m_ir_i),;g_1,...g_s\right),\left(-\rho-vs'-\sum_{i=1}^{k}(n_ir_i),;h_1,...h_s\right),(a_j:\alpha_{1k};...;\alpha_{sk})_{1,p}:(c_{1k},\gamma_{1k})1,p_1;...;(c_{sk},\gamma_{sk})_{1,P_S} \\ (b_j:\beta_{1k};...;\beta_{sk})_{1,q},\left(-\rho-\sigma-(u+v)s'-\sum_{i=1}^{k}(m_i+n_i)r_i-1:g_1+h_1;...;g_s+h_s\right):(d_{1k},\delta_{1k})1,q_1;...;(d_{sk},\delta_{sk})_{1,q_S} \end{matrix}\right] \quad (3.1)$$

Where

$$G(s') = \frac{\prod_{l=1}^{P}(a_l)_{s'}}{\prod_{l=1}^{Q}(b_l)_{s'}} \frac{Z^{s'}}{\Gamma(s'+1)}$$

Case- II. from the equation from (1.3d)

$$\int_{-1}^{1}(1-\mu)^{\rho}(1+\mu)^{\sigma} S_{n_1,....,n_k}^{m_1,....,m_k}\left[y_1(1-\mu)^{m_1}(1+\mu)^{n_1},...,y_k(1-\mu)^{m_k}(1+\mu)^{n_k}\right] E_{\varphi'}[z(1-\mu)^{u}(1+\mu)^{v}]$$

$$H_{p,q:p_1q_1;....;p_s,q_s}^{o,n:m_1,n_1;..;m_s,n_s}\left[\begin{matrix} z_1(1-\mu)^{g_1}(1+\mu)^{h_1}, \\ \vdots \\ z_s(1-\mu)^{g_s}(1+\mu)^{h_s}, \end{matrix}\middle| \begin{matrix} (a_k:\alpha_{1k};...;\alpha_{sk})_{1,p}:(c_{1k},\gamma_{1k})1,p_1;...;(c_{sk},\gamma_{sk})_{1,P_S} \\ (b_k:\beta_{1k},...;\beta_{sk})_{1,q}:(d_{1k},\delta_{1k})1,q_1;...;(d_{sk},\delta_{sk})_{1,q_S} \end{matrix}\right] d\mu$$

$$= 2^{\rho+\sigma+1} \sum_{r_1=0}^{[n_1/m_1]},...,\sum_{r_k=0}^{[n_k/m_k]} (-n_1)_{m_1r_1},...,(-n_k)_{m_kr_k} A(n_1,r_1;...;n_k,r_k) \prod_{i=1}^{k} \frac{y_i^{r_i}}{r_i!} 2^{(u+v)s'} \sum_{s'}^{\infty} T(s') 2^{\sum_{i=1}^{s}(m_i+n_i)r_i}$$

$$H_{p+2,q+1}^{0,n+2}\left[\begin{matrix} z_1 2^{(g_1+h_1)\lambda_1} \\ \vdots \\ z_s 2^{(g_s+h_s)\lambda_s} \end{matrix}\middle| \begin{matrix} \left(-\sigma-us'-\sum_{i=1}^{k}(m_ir_i),;g_1,...g_s\right),\left(-\rho-vs'-\sum_{i=1}^{k}(n_ir_i),;h_1,...h_s\right),(a_j;\alpha_{1k};...;\alpha_{sk})_{1,p}:(c_{1k},\gamma_{1k})1,p_1;...;(c_{sk},\gamma_{sk})_{1,P_S} \\ (b_j:\beta_{1k};...;\beta_{sk})_{1,q},\left(-\rho-\sigma-(u+v)s'-\sum_{i=1}^{k}(m_i+n_i)r_i-1:g_1+h_1;...;g_s+h_s\right):(d_{1k},\delta_{1k})1,q_1;...;(d_{sk},\delta_{sk})_{1,q_s} \end{matrix}\right] \quad (3.2)$$

Case –III First of all the second class of polynomials convert to the first class in (2.3) , after which convert $S_{n_1,....,n_k}^{m_1,....,m_k}[y_1,...,y_k]$ in to $S_n^m[y]$ then the multivariable polynomial is found in the form of k in first class of polynomial i.e $A(n_1,r_1;...;n_k,r_k) = A_1(n_1,r_1);...;A_k(n_k,r_k)$ as well as in (2.3) we take m, n, p, q =0 , $\alpha_{ik},\beta_{ik},c_{ik},d_{ik}=1$ and (1.3d) we acquire

$$\int_0^{\infty}\left[\left(az+\frac{b}{z}\right)^2+d\right]^{-\vartheta-1}\prod_{i=1}^{k} S_{n_i}^{m_i}\left[y_i\left(az+\frac{b}{z}\right)^2+d\right]^{-mi} \times H_{p',q'+1}^{m'+1,n'}\left[\left[a_0\left(az+\frac{b}{z}\right)^2+d\right]^{-\gamma}\middle|\begin{matrix}(r_k,R_k)1,p\\(l_0,L_0),(l_k,L_k)1,q\end{matrix}\right]E\varphi[z]$$

$$H_{p_1,q_1}^{m_1\ n_1}\left[\left[z_1\left(az+\frac{b}{z}\right)^2+d\right]^{-s_1},...,H_{p_s,q_s}^{m_s\ n_s}\left[z_s\left(az+\frac{b}{z}\right)^2+d\right]^{-s_s}\right]dz \quad (3.3)$$

$$= \frac{\sqrt{\pi}\, a_0^{-\gamma}}{2aL_0(4ab+d)^{\upsilon+\gamma+\eta_G s'+\frac{1}{2}}}\prod_{i=1}^{k}\sum_{r=1}^{[n_k/m_k]}\sum_{s=0}^{\infty}\sum_{s'=0}^{\infty}(-n_i)_{m_ir_i}A_i(n_i,r_i)\frac{y_i^{r_i}(4ab+d)^{-\sum_{i=1}^{k}m_ir_i}}{r_i!}T(s')\rho_s M(s)$$

$$H_{1,1:D}^{0,1:A}\left[\begin{matrix} z_1(4ab+d)^{-s_1} \\ \vdots \\ z_s(4ab+d)^{-s_s} \end{matrix}\middle|\begin{matrix}\left(½-\vartheta-\gamma-\eta_g s'+\sum_{i=1}^{k}m_ik_i;s_1,...,s_s\right),E:V \\ \left(-\vartheta-\gamma-\eta_g s'+\sum_{i=1}^{k}m_ik_i;s_1,...,s_s\right),F:W\end{matrix}\right]$$

See $T(s')$ in (1.3e)

Case- IV Provided the position related to that of equation (2.3) with m’= n’=p’=q’=0 and

$-\gamma$ =1, L_0=1, $\alpha_{ik},\beta_{ik},c_{ik},d_{ik}=1$

$$\int_0^\infty \left[\left(az+\frac{b}{z}\right)^2+d\right]^{-\vartheta-1} S_{n_1,\ldots,n_k}^{m_1,\ldots,m_k}\left[y_1\left(az+\frac{b}{z}\right)^2+d\right]^{-m_1},\ldots,\left[y_k\left(az+\frac{b}{z}\right)^2+d\right]^{-m_k}\times$$

$$H_{0,1}^{1,0}\left[\left[a_0\left(az+\frac{b}{z}\right)^2+d\right]\Big|_{(l_0,1),(l_k,L_k)1,q}^{(r_k,R_k)1,p}\right]{}_PF_Q[z]\,G\left[\left[z_1\left(az+\frac{b}{z}\right)^2+d\right]^{-s_1},\ldots,\left[z_s\left(az+\frac{b}{z}\right)^2+d\right]^{-s_s}\right]dz\,.$$

$$=\frac{\sqrt{\pi}\,a_0}{2a(4ab+d)^{\upsilon+\eta_G s'+\frac{3}{2}}}\sum_{r_1=0}^{[n_1/m_1]},\ldots,\sum_{r_k=0}^{[n_k/m_k]}\sum_{s=0}^{\infty}\sum_{s'=0}^{\infty}(-n_1)_{m_1r_1},\ldots,(-n_k)_{m_kr_k}A(n_1,r_1;\ldots;n_k,r_k)\ M(s)\rho_s G(s')$$

$$\prod_{i=1}^{k}\frac{y_i^{r_i}(4ab+d)^{-\sum_{i=1}^{k}m_ir_i}}{r_i!}H_{p+1,q+1:D}^{0,n+1:A}\left[\begin{matrix}z_1(4ab+d)^{-s_1}\\ \vdots\\ z_s(4ab+d)^{-s_s}\end{matrix}\left|\begin{matrix}\left(-\vartheta-\eta_g s'+\sum_{i=1}^{k}m_ik_i;s_1,\ldots,s_s\right),E:V\\ \left(-\vartheta-\eta_g s'-1+\sum_{i=1}^{k}m_ik_i;s_1,\ldots,s_s\right),\ F:W\end{matrix}\right.\right] \quad (3.4)$$

see G(s') in (1.3c)

IV. CONCLUSION

The H-function of multivariable and M-series described with various polynomials in research paper. This work relatively basic in nature, as a result of specifying of the parameters on this function. We would find another special function like as Major's G-function, Mac-rodent's, Bessel function, Wright's function, Fox's H-function, generalized hypergeometric function and furthermore on the specializing the parameters of this function obtain various special cases.

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