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On Nano Generalized Semi Maximal Closed Sets in Nano Topological Spaces

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Abstract: The purpose of this paper is to define and study a new class of sets called Nano maximal closed sets and Nano generalized semi maximal closed sets in Nano topological spaces. Basic properties of Nano maximal closed set and Nano generalized semi maximal closed set are also analyzed.

Keywords: Nano Maximal open sets, Nano Maximal closed sets, Nano generalized semi maximal closed sets.

I. INTRODUCTION

In 1970 N. Levine[7] introduced generalized closed sets in topological spaces. In 1990, Arya et al.,[1] have introduced the concept of generalized semi closed sets. Rough set theory proposed by Pawlak[8] is a new mathematical tool for data reasoning. The basic structure of rough set is an approximation space such as lower, upper and boundary approximations. The rough set satisfies the topological conditions is called Nano topology and it was introduced by Leills Thivagar[6]. Bhuvaneswari et al.,[3] have introduced and investigate Nano generalized semi closed sets. In 2003 maximal open set was introduced and studied by F. Nakaoka et al.,[5]. And in 2006,"Minimal closed and maximal closed sets" were discussed by F. Nakaoka et al.,[4]. In this paper, a new class of sets called Nano maximal closed sets and Nano generalized semi maximal closed sets in Nano topological spaces are introduced and also studied some of its properties.

II. PRELIMINARIES

Definition 2.1[8]: Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as indiscernibility relation Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U,R) is said to be the approximation space. Let $X \subseteq U$.

- The lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and its is denoted by $U_R(X)$. That is, $L_R(X) = \bigcup_{x \in U} \{R(x) : R(x) \subseteq X\}$, R(x) denotes the equivalence class determined by x.
- The upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and it is denoted by $U_R(X)$. That is $U_R(X) = \bigcup_{x \in U} \{R(x) : R(x) \cap X \neq \phi\}$
- The boundary region of X with respect to R is the set of all objects, which can be classified neither as X nor as not-X with respect to R and it is denoted by $B_R(X) = U_R(X) - L_R(X)$

Definition 2.2[6]: Let U be the universe, R be an equivalence relation on U and $\tau_R(X) = \{U, \phi, L_R(X), U_R(X), B_R(X)\}$, where $X \subseteq U$ which satisfies the following axioms:

- U and $\Phi \in \tau_R(X)$.
- The union of the elements of any sub-collection of $\tau_R(X)$ is in $\tau_R(X)$.
- The intersection of the elements of any finite sub collection of $\tau_R(X)$ is in $\tau_R(X)$.

That is, $\tau_R(X)$ is a topology on U called the Nano topology on U with respect to X. We call $(U, \tau_R(X))$ as the Nano topological space. The elements of $\tau_R(X)$ are called Nano open sets.

Definition 2.3[6]: A subset A of a Nano topological space $(U, \tau_R(X))$ is called a Nano semi-open set if $A \subseteq Ncl[NInt(A)]$. The complement of a Nano semi open set of a space U is called Nano semi closed set in U.

Definition 2.4[3]: A Nano semi-closure of a subset A of $(U, \tau_R(X))$ is defined as the intersection of all Nano semi closed sets containing A and it is denoted by $Nscl(A)$. $Nscl(A)$ is the smallest Nano semi closed set containing A.

Definition 2.5[3]: A subset A of $(U, \tau_R(X))$ is called a Nano generalized semi closed set (briefly Ngs closed) if $Nscl(A) \subseteq V$ whenever $A \subseteq V$ and V is Nano open set in $(U, \tau_R(X))$.

Definition 2.6[5]: A non-empty open set U of a topological space X is said to be a maximal open (resp. maximal closed) set if and only if any open (resp. closed) set which contains A, is either A or X.

Definition 2.7: A subset A of a topological space X is said to be a generalized maximal closed set if $cl(A) \subseteq U$ where $A \subseteq U$ and U is maximal open set in X.

Definition 2.8: A subset A of a topological space (X,τ) is said to be generalized semi maximal closed set if $Scl(A) \subseteq U$ whenever $A \subseteq U$ and U is maximal open in U.

III. NANO GENERALIZED SEMI MAXIMAL CLOSED SETS

In this section, the definition Nano maximal closed sets and Nano generalized semi maximal closed sets are introduced and studied some of its properties.

Definition 3.1: A non-empty subset A of a Nano topological space $(U,\tau_R(X))$ is said to be Nano maximal open set (resp. Nano maximal closed set) if and only if any Nano open(resp. Nano closed) set which contains A is U or A.

Example 3.2: Let $U=\{a,b,c,d\}$ with $U/R=\{\{b\},\{d\},\{a,c\}\}$ and $X=\{a,b\}$

Then $\tau_R(X)=\{U,\phi,\{b\},\{a,c\},\{a,b,c\}\}$ which are Nano open sets.

The Nano closed sets = $\{U,\phi,\{d\},\{b,d\},\{a,c,d\}\}$

The Nano maximal open sets $=\{\{a,b,c\}\}$

The Nano maximal closed set = $\{\{b,d\},\{a,c,d\}\}$

The Nano semi closed sets = $\{U,\phi,\{b\},\{d\},\{a,c\},\{b,d\},\{a,c,d\}\}$

The Nano generalized semi closed sets are

$\{U,\phi,\{a\},\{b\},\{c\},\{d\},\{a,c\},\{a,d\},\{b,d\},\{c,d\},\{a,c,d\},\{a,b,d\},\{b,c,d\}\}$

Theorem 3.3: For any Nano topological space U, the following statements are true:

(i). Let A be a Nano maximal open set (resp. Nano maximal closed set) and B an Nano open set (resp. Nano closed set). Then, $A\cup B=U$ or $B\subset A$.

(ii). Let A and C be Nano maximal open sets (resp. Nano maximal closed set). Then, $A\cup C=U$ or $A=C$.

Proof: (i) Let B be an Nano open set (resp. Nano closed set) such that $A\cup B\neq U$. Since A is a Nano maximal open set (resp. Nano maximal closed set) and $A\subset A\cup B$, which implies that $A\cup B=A$. Therefore $B\subset A$.

(ii) If $A\cup C\neq U$, then $A\subset C$ and $C\subset A$ by (i). Therefore $A=C$.

Theorem 3.4: Let $(U,\tau_R(X))$ be a Nano topological space and A be a subset of $(U,\tau_R(X))$. If A is a Nano maximal closed set, then it is a Nano generalized semi closed set.

Proof: Let A be a Nano maximal closed set. But every Nano maximal closed set is a Nano closed set and every Nano closed set is a Nano generalized semi closed set and hence A is a Nano generalized semi closed set.

Remark 3.5: The converse of the above Theorem 3.4 need not be true. From the Example 3.2, {a} is a Nano generalized-semi closed set which is not a Nano maximal closed set

Definition 3.6: A subset A of a Nano topological space $(U,\tau_R(X))$ is said to be Nano generalized semi maximal closed set if $Nscl(A)\subseteq V$ whenever $A\subseteq V$ and V is Nano maximal open in $(U,\tau_R(X))$.

Example 3.7: By the Example 3.2, the Nano generalized semi maximal closed sets are $\{\phi,\{a\},\{b\},\{c\},\{a,c\}\}$

Theorem 3.8: Let $(U,\tau_R(X))$ be a Nano topological space and A be a subset of $(U,\tau_R(X))$ If A is a Nano generalized semi maximal closed set, then it is a Nano generalized semi closed set in $(U,\tau_R(X))$.

Proof: Let A be a Nano generalized semi maximal closed set. By the definition of Nano generalized semi maximal closed set, $Nscl(A)\subseteq V$ whenever $A\subseteq V$ and V is Nano maximal open in $(U,\tau_R(X))$. Since every Nano maximal open set is a Nano open set, which implies that V is a Nano open set. Thus, $Nscl(A)\subseteq V$ whenever $A\subseteq V$ and V is Nano open set in $(U,\tau_R(X))$. Hence A is Nano generalized semi closed set.

Remark 3.9: The converse of the above Theorem 3.8 need not be true. From the Example 3.7, {d} is a Nano generalized semi closed set which is not a Nano generalized semi maximal closed set.

Theorem 3.10: Let $(U,\tau_R(X))$ be a Nano topological space and A and B are subsets of $(U,\tau_R(X))$. If A and B are Nano generalized semi maximal closed sets in $(U,\tau_R(X))$ then the intersection of A and B is also a

Nano generalized semi maximal closed set in $(U, \tau_R(X))$.

Proof: Let A and B be two Nano generalized semi maximal closed sets in $(U, \tau_R(X))$. Let V be a Nano maximal open in $(U, \tau_R(X))$ such that $A \subseteq V$ and $B \subseteq V$, which implies that $A \cap B \subseteq V$. As A and B are Nano generalized semi maximal closed sets in $(U, \tau_R(X))$, $Nscl(A) \subseteq V$ and $Nscl(B) \subseteq V$. Now, $Nscl(A \cap B) = Nscl(A) \cap Nscl(B) \subseteq V$. Thus we have, $Nscl(A \cap B) \subseteq V$ whenever $A \cap B \subseteq V$ and V is Nano maximal open set in $(U, \tau_R(X))$ which implies that $A \cap B$ is a Nano generalized semi maximal closed set.

Remark 3.11: The union of two Nano generalized semi maximal closed sets need not be Nano generalized semi maximal closed set. From the Example 3.7, Let $A = \{a\}$ and $B = \{b\}$. Now, $A \cup B = \{a\} \cup \{b\} = \{a, b\}$ which is not a Nano generalized semi maximal closed set.

Theorem 3.12: Let A be a Nano generalized semi maximal closed set in a Nano topological Space $(U, \tau_R(X))$. If $A \subseteq B \subseteq Nscl(A)$, then B is also a Nano generalized semi maximal closed subset of $(U, \tau_R(X))$.

Proof: Let V be a Nano maximal open set of a Nano generalized semi maximal closed subset of $(U, \tau_R(X))$ such that $B \subseteq V$. As $A \subseteq B$, we have $A \subseteq V$. As A is Nano generalized semi maximal closed set, $Nscl(A) \subseteq V$. Given that $B \subseteq Nscl(A)$, which implies that $Nscl(B) \subseteq Nscl(A)$. But $Nscl(A) \subseteq V$, which implies that $Nscl(B) \subseteq V$ whenever $B \subseteq V$ and V is Nano maximal open in $(U, \tau_R(X))$. Hence, B is a Nano generalized semi maximal closed subset of $(U, \tau_R(X))$.

Theorem 3.13: If A is a Nano generalized semi maximal closed set in a Nano topological space $(U, \tau_R(X))$, then for each $x \in Nscl(A)$, $Nscl\{x\} \cap A \neq \phi$.

Proof: Let A be any Nano generalized semi maximal closed set in $(U, \tau_R(X))$ such that for each $x \in Nscl(A)$. Let $Nscl\{x\} \cap A = \phi$ Then $A \subseteq [Nscl\{x\}]^c$, where $[Nscl\{x\}]^c$ is an Nano open set in $(U, \tau_R(X))$. By the Theorem 3.8, A is Nano generalized semi closed set and we have $A \subseteq [Nscl\{x\}]^c$. Thus, $Nscl(A) \subseteq [Nscl\{x\}]^c$, which is a contradiction to the fact that $x \in Nscl(A)$. Therefore $Nscl\{x\} \cap A \neq \phi$.

Figure 3.1 follows from the above results

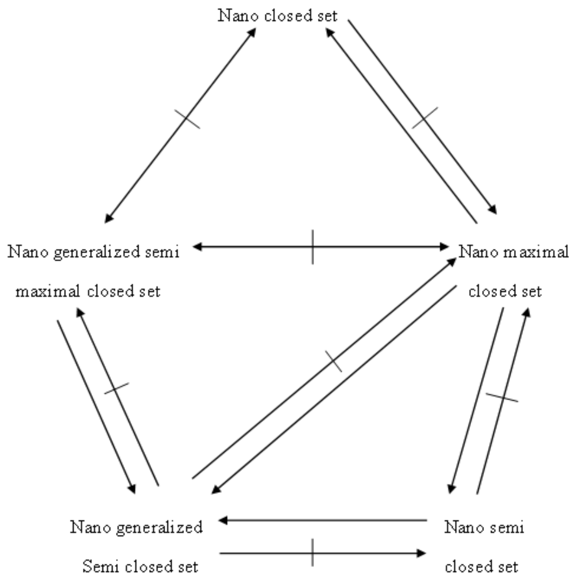

Fig. 3.1

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